TRANSPORT AND INFRASTRUCTURE COUNCIL

Australian Transport Assessment and Planning Guidelines

Road Reliability Measurement –Research Report

November 2022

This publication was prepared by the Australian Transport Assessment and Planning (ATAP) Steering Committee and approved by the Transport and Infrastructure Senior Officials' Committee. The Committees cannot accept responsibility for any consequences arising from the use of this information. Readers should rely on their own skill and judgment when applying any information or analysis to particular issues or circumstances.

ISBN XXXXXX

November 2022

For enquiries or feedback about this publication please contact:

ATAP Steering Committee Secretariat
Australian Transport Assessment and Planning Guidelines
Commonwealth Department of Infrastructure, Regional Development and Cities
GPO Box 594 CANBERRA ACT 2601
Email: atap@infrastructure.gov.au

This publication is available from the website atap.gov.au

Contents

List of Figures

List of Tables

List of Abbreviations

Abbreviation	Full Name
ARSD	Approximated route standard deviation
ATAP	Australian transport assessment and planning
BPR	Bureau of public roads
CAH	Controlled access highways
CBA	Cost–benefit analysis
CCM	Correlation coefficient model
CI	Congestion index
CoV	Coefficient of variation
CRM	Correlation route model
DM	Dutch model
ECVM	Exponential coefficient of variation model
GEH	Geoffrey E. Havers
GMA	Greater metropolitan area (of Sydney)
LLM	Log-linear model
LM	Linear model
LSCM	Length standardised cubic model
LSLM	Length standardised linear model
MRWA	Main roads Western Australia
NetPReS	Network performance reporting system
NPI	National performance indicators
NZM	New Zealand model
OD	Origin-destination
PMDM-1	Power mean delay model
PMDM-2	Polynomial mean delay model
RMSE	Root mean squared error
SD	Standard deviation
SP	Stated preference
StrUE	Strategic user equilibrium
TMR	Department of Transport and Main Roads
UE	User equilibrium
UKM	United Kingdom model
URM	Unified reliability model
V/C	Volume-to-capacity ratio
VoR	Value of reliability
WTP	Willingness to pay

Executive Summary

Background

This research report presents the findings of Project TAP6234, which was to develop an approach for measurement of road reliability for inclusion in the Australian Transport Assessment and Planning (ATAP) Guidelines. It is widely recognised that travellers consider travel time reliability in their travel decision making. Therefore, the benefits of improved travel time reliability should be accounted for in appraisal of transport related initiatives. The work in this project was one of two parts of an initiative aimed at developing a methodology and parameters for calculating the benefits of improvements in travel time reliability measured as follows:

Value of a travel time reliability improvement benefit ($) = Unit value of reliability ($/min) x Saving in travel time variability (mins)

For cost–benefit analysis (CBA) purposes, estimates of reliability throughout a network or along a corridor are required for the base case (without the initiative) and the project case (with the initiative) and the benefit is calculated from the difference between them (the saving in travel time variability). This project relates to the second part of the benefit formula which is the reduction in travel time variability.

A range of Australian data for observed travel times and their standard deviations was used to refine formulas that can be applied to a range of road stereotypes (capacity and congestion). This project built upon the work already undertaken by Austroads, academia and internationally.

This project's purpose was to obtain calibrated formulas that relate the standard deviation of travel times to observed measures of road capacity and congestion to model travel time variability in Australia. The project commenced with a literature review of best practice methods of forecasting changes in travel time variability internationally and in Australia as well as how variability is treated in strategic transport models. Data on travel time variability for different roads and journeys in different cities and regions were collected and analysed, alternative functional forms tested, and the best fitting functional form(s) were recommended. These recommended models were then tested on two actual past road infrastructure projects and one test involved the use of an urban transport model. Recommendations were developed for a practical approach to estimate changes in travel time variability for use in the CBA of road initiatives and for use in transport models.

Purpose of the work

In the context of transport, travel time reliability is used to describe how certain the travel time is for a journey for a road user. Poor travel time reliability causes people to leave earlier to reduce the probability of being late and to be late more often. Reliability can influence route choice decisions including whether to use a toll road and is therefore relevant for toll road patronage forecasting. Travel time reliability has been an active area of research in the past one decade owing to its repercussions on traffic movement and congestion in a road network. The benefit of changes in travel time reliability expressed as a monetary amount ought to be counted in CBAs of transport projects and/or policy changes. The benefit is estimated by multiplying the predicted changes in travel time reliability improvements for a project by road users' willingness-to-pay (WTP) (measured in $/h) to reduce travel time variability. This project develops a methodology and parameter values to estimate the former quantity, that is, a way to predict changes in travel time reliability. Determination of WTP values of reliability changes is beyond the scope of this work. The mathematical models developed to forecast travel time reliability for different elements in a road network need to be readily usable by practitioners. While parameter values estimated from available data are provided, practitioners should be able to recalibrate the models using their own data.

Using Standard Deviation of Travel Time

Travel time reliability relates to the distribution, spread or dispersion of travel times over a link or route and over time. It can be measured in a variety of ways. For this project, the chosen measure is the standard deviation (SD) of travel time over a given period. This was recommended for practical application in CBAs and toll road patronage forecasting by a scoping study (TIC 2016). The main two approaches that have been developed to value reliability are the mean-variance and scheduling approaches. The mean-variance approach yields a single value as the marginal value of one SD of travel time. It assumes symmetric penalties for being early and being late. The scheduling approach attaches different values to time early and time late. While the scheduling approach is conceptually preferred, it is quite complex to apply because it demands knowledge of travellers' preferred arrival times. The mean-variance approach is the simplest and most feasible way to address reliability in CBAs of road projects.

The Three Levels of Analyses

As the road network is composed of several elements, this project addresses computing travel time variability at the link, route and the entire network level.

- Links are the basic component in a road network and are subjected to day-to-day changes in travel time. Thus, it is essential to predict SD of link travel time which can then contribute to understanding macroscopic elements in a road network.

- A route is a series of links traversed by users to travel from an origin to a destination. It is paramount to study travel time variability forecasting at a route level because transport users experience and make decisions based on travel time variability for entire journeys, and not for the individual links that comprise a route. Summing the travel time SDs of the links that constitute a route is not a correct way to obtain route travel time SD. It is mathematically flawed as it assumes that the constituent links are independent of one another, that is the traffic condition on one link does not affect other links. The traffic conditions on individual links are correlated. For example, heavy congestion can result in a queue-spillback. The congested traffic condition gradually expands to the upstream links making the links interdependent. Thus, this project investigates the appropriate method to predict route travel time SD to account for interdependencies between link SDs.

- Network level analysis of travel time reliability has also been considered as it provides a tool to evaluate travel time reliability benefits/disbenefits across an entire road network, providing useful information for transport planning and appraisal of large projects with widespread affects throughout a network.

Model Development

An extensive literature review was undertaken at the start of the project to determine the state-of-the-art in modelling travel time reliability for links, routes and a network level.

Link — For the link level analysis, the literature search identified 11 models for determining travel time variability. As the models were developed in different geographies, a numerical experiment was used as a common test bed to compare these models. Most of these models were considered unsuitable as they forecasted an ever-increasing travel time SD as congestion increased, which is unrealistic as travel time tends to stabilise at higher congestion levels. As a result, three models, the UK model, New Zealand model and the Dutch model, were shortlisted for further consideration. Furthermore, an Australian Transport Assessment and Planning (ATAP) model was developed.

The ATAP link model utilises a nonlinear equation expressing travel time coefficient of variation (CoV) in as a function of the congestion index (CI; defined as the ratio of prevailing travel time and free-flow travel time) on a link. CoV was selected as the dependent variable, over SD, because CoV represents a standardised measure that facilitates comparison between links of varying lengths. Similarly, CI was chosen over volume-to-capacity ratio because volume, which represents demand, is not easily measurable. For example, while traffic volume is equal to demand in under-saturated conditions, it is much lower than the demand in saturated traffic. Travel time on the other hand is consistent in both under-saturated and saturated conditions. Furthermore, link capacities are also difficult to measure. Hence, the CI approach has advantages over the volume-based approach.

Unlike the other three shortlisted models, the ATAP link model curve increases sharply at lower CIs, and follows a declining growth rate at higher CIs. This trend is consistent with real-world traffic dynamics where the improvements in travel time reliability are significant at lower congestion levels and miniscule at higher congestion levels. Thus, the ATAP link model (shown below) was chosen as the recommended approach for forecasting link travel time variability.

$$\boldsymbol{CoV = a\left[\frac{(CI-1)}{CI}\right]^{b} \ \forall\ CI \geq 1}$$

Where:
CoV = coefficient of variation, $\frac{\sigma}{T}$
CI = congestion index = $max\ (1, \frac{T}{T_f})$, where T = mean travel time (minutes) and T_f = free-flow travel time (minutes)
a = calibration parameter that sets the upper limit of CoV, $a \mid a > 0$
b = calibration parameter that determines the rate at which CoV approaches the maximum, $b \mid b \in (0, 1)$

Route — For the route level analysis, the literature search identified the Nicholson Model (referred to as the correlation route model (CRM)) as the recommended approach to determine route travel time variability. The model not only adds the individual travel time variances of the constituent links, but also the travel time correlations among links. Thus, the CRM (shown below) is able to capture link inter-dependencies in forecasting route travel time SD (unlike simply adding individual link travel time SDs).

$$\boldsymbol{\sigma_r^2 = \sum_{i=1}^{n} \sigma_i^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \rho_{i,j}\sigma_i\sigma_j\ , i < j}$$

Where:
σ_r^2 = variance of travel time of route with n number of links
σ_i^2 = variance of travel time of link i
σ_i = SD of travel time of link i
σ_j = SD of travel time of link j
$\rho_{i,j}$ = correlation coefficient of travel time between links i and j

The CRM comprises two sub-models: 1) the ATAP link model that determines travel time SD of individual links, and 2) the correlation coefficient model (CCM) that predicts the degree of correlation among links forming a given route. The CCM (shown below) is a linear-log model relating the degree of correlation to the log of distance between the mid-points of two links within a route.

$$\boldsymbol{\rho_{i,j} = Max[0, a.\mathrm{Ln}(\mathrm{L}) + \mathrm{b}]}$$

Where:
$\rho_{i,j}$ = correlation coefficient of travel time between links i and j where i < j
L = distance between the midpoints of two links (kilometres)
a, b = parameters

Network — For network-level analysis, two methods of determining travel time reliability were identified. The first method, the Approximate Route Standard Deviation (ARSD), determines the route travel time variability as a correction factor times the summation of link travel time SDs forming the route. The correction factor roughly accounts for the errors arising due to not considering link interdependencies. While this method cannot provide an accurate route travel time SD, it is a practical and straightforward approach to obtain an approximate value rather than using the CRM, which is more complex and resource intensive.

$$\boldsymbol{\sigma_r \approx \gamma \sum \sigma_l}$$

Where:
σ_r = SD of travel time on route
σ_l = SD of travel time on links

γ = correction factor

It should be noted that all of the above models only focus on predicting the travel time variability and do not take into consideration its impact on the resulting route choice behaviour of road users. This gap is filled by the second approach, the strategic user equilibrium (StrUE), which is a novel methodology. It is much superior to ARSD because it accounts for impacts of travel time variability on route choice and vice-versa. StrUE estimates travel time variability given day-to-day changes in origin–destination demands and/or link capacities. However, while methodologically correct, StrUE is more complex and resource intensive to implement.

Calibration and Validation

Datasets — The data for this exercise was obtained from different jurisdictions across Australia: Perth (provided by MRWA), Gold Coast and Brisbane (provided by TMR) and Sydney (available from UNSW). The Perth network performance reporting system (NetPReS) dataset used is hybrid traffic data for 29 arterial and freeway routes in the Perth metropolitan area collected over 2018 to 2020. The data include traffic speed for every 15-minute period during the day for each link. The NetPReS dataset was used to calibrate the ATAP link, CCM and the ARSD models. The Queensland data comprised daily NPI and Bluetooth data (by 15-minute periods) for arterials and freeways, and the Sydney data comprised travel time information for around 35 routes in Sydney collected using the Google API. Besides calibration and validation, this project tested two case studies (for Gold Coast and Perth) to assess the goodness of the CRM. This was considered desirable because the CRM includes the ATAP link and CCM as sub-models.

Link — Individual ATAP link models were calibrated for forecasting travel time reliability for arterial and freeway links. Four months of traffic data (that is August to November 2017) was used to calibrate the model. The free-flow speed was taken as the 99th percentile of all speed values (in 15-minute periods) witnessed across all weekdays, excluding public holidays, in a month. All the speeds were then converted into travel times using link length information. The data were then filtered to remove any observations from which data were missing or where the CI was less than 1 because travel time should always be greater than or equal to free-flow travel time. The CoV and CI variables were then transformed by taking logarithms, which were then used for calibration using the linear regression tool in MS-Excel. An alternative functional form was also tested alongside the ATAP link model. The alternative functional form produced a marginally better fit than the ATAP link model but was rejected because it is more difficult to calibrate, requiring non-linear regression, and the curve flattens out too much at high congestion levels. The calibrated ATAP model was then applied to Queensland and Sydney data for validation. It was found that the calibrated ATAP model showed a reasonably good fit to these data.

Route — To develop the CRM, the CCM was calibrated first using the NetPReS data. A separate CCM was calibrated for the following categories: (1) freeway and arterial, (2) inbound and outbound, and (3) AM (7-9am), Inter-peak (9am-3pm), PM (3-6pm) and Off-peak (5-7am and 6-9pm). Thus, there were 16 sets of calibration parameters. The ATAP link model together with the CCM were then utilised to determine the estimated route travel time SD. It was compared against the measured route travel time SD, which was determined as follows: 1) summing up the individual link travel times for a given 15-minute time period across all weekdays, excluding public holidays, in the month and then finding its SD. The comparison revealed that the CRM gives a reasonable fit to the measured route travel time SDs. Case studies were undertaken using before-vs-after data for infrastructure improvements in Brisbane and Perth. For the Perth case study, traffic data for four months in 2018 (before) was compared against the same four months in 2019 (after). The results again substantiated the accuracy of CRM in predicting observed route travel time SD. However, the results from the Brisbane case study were not as promising, which could be explained due to a lack of localised calibration (that is the models were calibrated on Perth data and not on Brisbane data) and geographical differences in driving behaviour, road design and traffic conditions.

Network — For the network-level analysis, the ARSD method was calibrated using the NetPReS data. ARSD model calibration was undertaken for each arterial and freeway to obtain the parameter value. The calibrated ARSD model was then used on the Sydney data to estimate route travel time SD. The case study tested three scenarios such as an increase in roadway capacity. The results showed predicted changes in route travel time SDs for the different arterial and freeway routes in Sydney. Then, the StrUE model was applied on the same Sydney network and same three scenarios tested. The differences between ARSD and StrUE along with their merits and demerits were discussed.

Benefits of this Work

This report presents the development and implementation of robust link and route-level models to predict travel time reliability in a road network — the ATAP link model and the CRM route model. Two different methods for modelling network travel time reliability, ARSD and StrUE, were also shown. The report's contribution is to allow the ATAP guidelines to present methodologies and parameters for evaluating travel time variability.

While the formulas have been rigorously calibrated and validated to provide default values, it is recommended that practitioners calibrate the models using their own local data following the calibration processes outlined in this report to account for traffic dynamics characteristic to a specific geography or jurisdiction.

1 Introduction

1.1 Project Objectives

The objective of this project is to obtain calibrated formula(s) related to the standard deviation (SD) of travel times to observed measures such as road capacity and level of congestion in order to determine travel time variability in Australia (ATAP Brief, 2019a). The following tasks were set to achieve the objectives of the project:

1. Conduct a literature review on the state-of-the-art for forecasting changes in travel time variability internationally and in Australia and how it is considered in strategic transport models[1]. The objective has been addressed in Section 2.
2. Collect travel time variability data for different roads and journeys across different cities and regions in Australia. Details of the available data are presented in Appendix A3.
3. Perform statistical analyses on the available datasets testing different functional forms and find the best fitting functional form. Sections 3 and 4 discuss the data analysis methodology and results respectively.
4. Test the recommended approaches on at least one current or past road infrastructure projects, which includes a test case showing the application of travel time variability in a strategic transport model. The objective has been addressed in Section 4 using Perth and Brisbane case studies. Additionally, Section 4 discusses the application of ARSD and StrUE on a Sydney case study.
5. Develop recommendations for practitioners to determine travel time variability, based on SD and coefficient of variation (CoV), which in turn feeds into travel time reliability estimation for application in project appraisal processes of road initiatives (e.g. cost benefit analysis (CBA)) and for use in transport models. It is noteworthy that while the integration of travel time reliability estimation into metropolitan strategic transport models is the ultimate objective, the aim of this project is to develop the formulas that can be used in the economic appraisal of projects.

1.2 Organisation of the Research Report

The organisation of this research report is as follows.

- Section 2 discusses the findings from the literature review on:

  (i) travel time variability estimation on links,

  (ii) approaches to determine travel time variability on routes, and

  (iii) introducing travel time reliability into the framework of strategic transport models.
- Section 3 presents the proposed methodology to be adopted at the three levels of interest, that is links, routes and network, based on literature review and empirical evidence.
- Section 4 details the calibration and validation procedure followed in model development and presents the formula(s), at link and route level, expressing travel time variability in terms of SD or CoV.
- Section 5 provides a summary of the study and the next steps towards the completion of this project.

This report also comprises an Appendix which details the work conducted during the entire duration of the project and how the proposed calibrated models were obtained.

[1] The review will not cover different approaches such as scheduling approaches for valuing reliability because the decision to use the mean-variance approach has already been taken (ATAP, 2019a; Transport and Infrastructure Council, 2016).

1.3 Scope of Work

ATAP (2019a) provides the formula to determine the valuation for travel time reliability which is shown in Equation 1.1.

$$\boldsymbol{TTRB} = \boldsymbol{v}.\Delta\boldsymbol{R} \quad \text{[EQ 1.1]}$$

Where:

TTRB: Travel time reliability benefit, $

v: unit value of travel time reliability, $/minutes

ΔR: Change in one SD in travel time, minutes

Equation 1.1 can be used for CBA to estimate and compare the monetary benefits of reliability changes over an entire network or along a corridor between the base case (without the initiative) and the proposed alternative (with the initiative)[2].

ATAP requires that the formulas developed be suitable for inclusion in the ATAP Guidelines and hence inclusion in economic assessments submitted to Infrastructure Australia and jurisdictional budget processes. Hence, this project looked at determining travel time variability at the three identified levels, that is links, routes and network. Furthermore, the formulas are calibrated using traffic data from three major urban centres in Australia. This is done because major cities, in general, experience frequent and widespread congestion which adversely impacts travel time variability. Considering regional road networks for analysis, which are less prone to congestion levels, is beyond the scope of this work.

[2] The first term in Equation 1.1 (the unit value of travel time reliability benefit) corresponds to the willingness to pay (WTP) of commuters, that is the dollar value per minute they associate towards an increased travel time reliability. This WTP measure can be determined through discrete choice experiments (e.g. stated preference surveys) where individuals are shown hypothetical scenarios regarding travel time variability (and hence reliability) on a travelled route. Determining the WTP estimate is beyond the scope of this report. This report focusses on the second term in Equation 1.1 (the change in one SD of travel time) by developing methodologies to determine travel time variability which eventually contributes to computing travel time reliability for a given alternative.

2 Literature Review

This section reviews the state-of-the-art in the field of travel time variability studies at a link and route level (Section 2.2 and Section 2.3 respectively). The section also discusses the approaches to include travel time reliability into the modelling framework of the strategic transport models and assess its implications on the network assignment (Section 2.4). Section 2.5 summarises the literature review by identifying appropriate methodologies from the several candidate approaches reviewed. The full literature review is attached as Appendix A1.

The main findings from the review are as follows:

Link Travel Time Variability

- The eleven link models reviewed have presented large variations in terms of model structures and parameters included, and their forecast travel time variabilities are inconsistent to a large degree, particularly when congestion levels are high.
- Travel time variability is dependent on a congestion index, link length and road types along a route. Travel time variability is directly related to the congestion index and is indirectly related to link length. Higher standard road types result in less travel time variability.
- Models that estimate the CoV are recommended (vis-à-vis SD of travel time).
- The congestion index is better represented as the ratio of mean travel time and free flow travel time (vis-à-vis V/C ratio).

Route Travel Time Variability

- Route travel time variability is best estimated by aggregating the link travel time variability, but the correlations of link travel time also need to be accounted for.

Network Travel Time Variability

- There are several proposed network assignment techniques that incorporate travel time reliability. While these techniques have been numerically demonstrated, their practicality for real-world application has not yet been tested. The difficulty arises from the issue that link SD of travel times are not additive and traditional techniques of network assignment rely on link costs to be additive.

2.1 Understanding Travel Time Variability and Reliability

Fosgerau et. al (2008) proposed that travel time (TT) is a summation of three constituent terms, namely the free flow time ($\mathrm{TT_{ff}}$), systematic delay ($\mathrm{TT_{sys\_delay}}$) and the unexplained delay ($\mathrm{TT_{unexp\_delay}}$). Equation 2.1 gives the expression to compute travel time on a given link in a network. The free flow travel time corresponds to the time under uninterrupted traffic flow on a link. Free flow travel time conditions are characterised by less traffic on link, generally observed between late nights and early mornings, where a driver can traverse the link cruising at free flow speed. Since the link length and free flow speed are defined for a link, the free flow travel time is also fixed for that link. The systematic delay ($\mathrm{TT_{sys\_delay}}$), which is additional travel time, occurs due to known sources of interrupted traffic flow such as presence of traffic lights, pedestrian crossings, and even systematic patterns in travel demand (e.g. lesser traffic on weekends in general). The unexplained delay ($\mathrm{TT_{unexp\_delay}}$) arises due to events such as traffic incidents and random demand fluctuations. It is the last two components, and the third component in particular, which heavily impacts on travel time to on a link across multiple days.

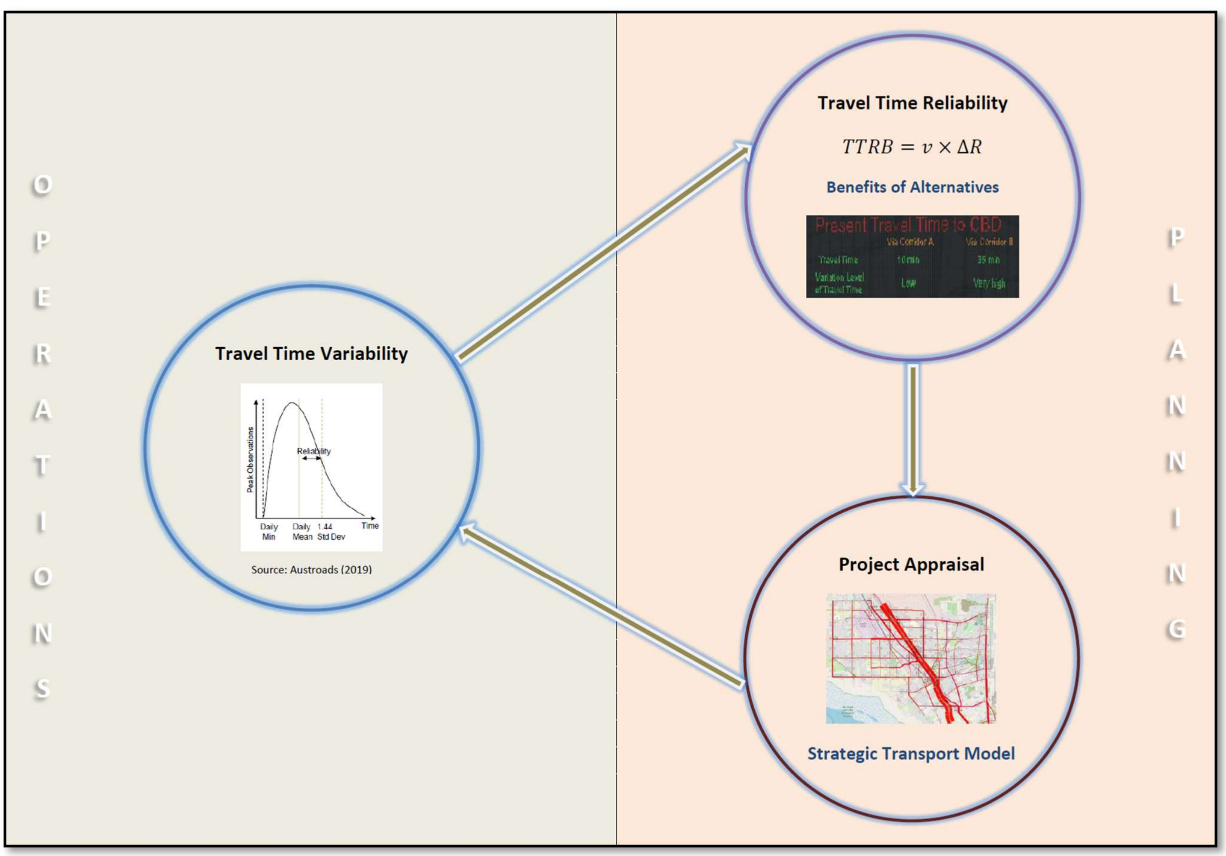
$$\boldsymbol{TT = TT_{ff} + TT_{sys\_delay} + TT_{unexp\_delay}} \quad \text{[EQ 2.1]}$$

Travel time variability has been defined as the distribution/spread or dispersion of travel times over a journey and over time (Osterle et al., 2017). Although a simple concept, MRWA (2016) and PIARC (2019) found that there is no global standard or industry agreed definition of variability. The frequency and magnitude of travel times are important considerations when evaluating travel time variability. According to Osterle et al. (2017), most travel time variability studies have investigated day-to-day changes in travel time and have expressed travel time variability as the random variation in travel time.

On the other hand, travel time reliability, as noted by Moylan et al. (2018), has been introduced using several definitions in the literature. In the context of transport, **travel time reliability** is used to describe how certain the travel time is on a journey. **Travel time variability** is a good measure of travel time reliability and is typically used by transport agencies. Travel time reliability is then calculated as a statistical measure using travel time variability (Austroads, 2011a).

Moylan et al. (2018) asserts that the terms of travel time reliability and travel time variability have, at times, been used interchangeably in past studies such as Batley et al. (2008) and Austroads (2011a). While the two terms are quite similar, there exist slight nuances between the two. **Travel time reliability** is the approximate consistency in travel times, for a defined road section or origin/destination, during a similar time-period under specific land use and road network settings. On the other hand, **travel time variability** represents the fluctuations in travel times (on a link or route) over a period of days. While achieving travel time reliability relates to the objective of users in planning their travels, travel time variability is the consideration of travel times in assisting their plans. In other words, travel time reliability is mainly used in the planning stage of a project while travel time variability either aids in or is a consequence of the former. Figure 2-1 shows the inter-relationship between the two concepts, that is travel time reliability and travel time variability. For a given state of the transport network, there could be travel time variations at link, route and network-wide levels. Such variability influences how individuals travel, and thus travel time reliability accounts for a monetary component in the economic appraisal process to identify the best transport project. Once, the project has been implemented, it again results in a new set of travel time variability, which ideally should be less than the before project implementation case.

Figure 2-1: Inter-relationship between travel time variability and reliability



2.1.1 Factors Affecting Travel Time Variability

There are several factors identified in the literature which affect travel time variability (Austroads, 2011a; Hyder et al., 2008). An understanding of these factors can help in understanding how a change in one factor affects the overall travel time variability. This in turn may be used in a forecasting model and/or to estimate the changes in reliability (as expressed in terms of variability) due a proposed project or policy on the road network. The factors are listed below:

- inadequate roadway capacity (resulting in a volume/capacity ratio (V/C) greater than 1)
- traffic incidents
- work zones and weather
- traffic control devices or traffic management systems
- special events
- vehicles stopping for non-traffic reasons
- demand fluctuations
- duration of peak period
- number of alternative routes available
- driver behaviour
- types of road users
- sharing road space
- on street parking
- variability over the survey period.

A study by Matsouki (2007), as quoted in Austroads (2011a), found that the main sources of travel time variability were: bottlenecks (40%), traffic incidents (25%), bad weather (15%), work zones (10%), poor signal timing (5%) and special events/others (5%). The above values can potentially be used for predicting the effect of interventions addressing these factors on variability. It is worth mentioning at this point that it is beyond the scope of this project to determine the individual contribution for each identified factor towards travel time variability.

The focus of this report is on developing a predictive model using road capacity and the related congestion effects on travel time variability (and hence reliability). The limitations or the lack of availability of the data would affect the ability to assess other applicable factors and the extents of applicability of the model. As far as practicable, effects that may affect significant demand fluctuations like public holidays and weekends were considered and controlled.

2.1.2 Measuring Travel Time Variability

Several models have been developed in the past to measure travel time variability (Austroads, 2011a; Moylan et al., 2018; MRWA, 2016; PIARC, 2019). The proposed models are as follows:

- Mean variance model
- Scheduling model
- Mean lateness model
- Options approach
- Vulnerability approach
- Other general models[3].

[3] For a full description see Section 4, Austroads (2011a)

A scoping study regarding valuing road travel time and reliability was undertaken to inform the ATAP Guidelines Steering Committee (TIC 2016). The study considered both the mean–variance and scheduling approaches and recommended the adoption of the mean–variance approach over the scheduling approach. The justification behind this recommendation was that the mean–variance approach results in a single unit value of reliability that represents the marginal value of one SD of travel time, whereas the scheduling approach requires separate values for being early and late. Although the scheduling approach is conceptually preferred in behavioural models that predict the disutility associated with travel time variability, the study noted that it would be more complicated to apply because it required knowledge of travellers' preferred arrival times.

This report has followed the scoping study's recommendation and adopts the mean–variance approach for practical applications in Cost Benefit Analysis (CBA) and toll road patronage forecasting. Applying this approach usually involves quantifying variability in terms of the SD of travel time.

Austroads (2011a) identifies different metrics to measure travel time variability for urban and rural road projects which have been used internationally. Moylan et al. (2018) extended this work by classifying these measures into three categories. Table 2-1: shows the classification of the metrics for travel time variability along with the description of each metric. The earlier scoping study recommended the travel time window metric, which measures variability as SD of travel time, as the basis of valuing travel time reliability (TIC 2016). Thus, this project aligns with the recommendation to develop an estimation method for determining the SD of travel time on a link and route level. Furthermore, this project also demonstrated the use of network modelling techniques to estimate the SD of travel time between an origin and destination.

Table 2-1: Measures of travel time variability

Category	Name	Description
Statistical range measures	Travel time window	Travel time window is the range of travel time to within one SD from mean travel time. Variability is measured as the SD of travel time.
	Coefficient of variation	Coefficient of variation is the ratio of SD and mean. The coefficient of variation is the fractional uncertainty in travel time.
	Variability index	Travel time variability is measured as the range of travel time which is calculated as the difference between the 95th and the 5th percentile travel time. Travel time variability index is the ratio of range during peak period and the range during the off-peak period.
	Width of travel time	Width of travel time is the ratio of the difference between the 90th percentile and 50th percentile travel time and the 50th percentile travel time.
Buffer measures	Buffer time	The buffer time is the amount of additional time which must be allocated to be 95% certain of achieving on-time arrival. Buffer time is the difference between the 95th percentile and the mean travel time.
	Buffer time index	The buffer time index is the ratio of the buffer time and the mean travel time.
	Planning time index	The planning time index is the ratio of the 95th percentile travel time and the free flow travel time.
	Misery index	Misery index is the ratio of the average of the 50th to 80th travel time and the mean travel time.
Tardy trip measures	Florida reliability statistic	The Florida reliability statistic is the probability that travel time is greater than the mean travel time.
	On-time arrival	On-time arrival is the proportion of trips that arrive within +10% of the mean travel time.
Skewness measures	Skewness statistic	Skewness statistic gives a measure of how extreme travel times can get as a measure of travel time reliability. Skewness statistic is equal to the ratio of the difference between the 90th percentile and 50th percentile travel time and the difference between the 50th percentile and 10th percentile travel time.

Source: Moylan et al. (2018)

2.2 Link Travel Time Variability

A link is a continuous section of the entire road segment which facilitates movement of vehicles and depicts homogeneous physical and traffic characteristics (e.g. discontinuities such as going from three lanes to two, presence of a signalised intersection on a straight road, and connection between on/off ramps to motorway are all represented as separate links).

Multiple links join to form a route connecting a given origin-destination pair which means that the travel time on a route is an aggregation of travel times on individual links. The literature review shows that several models have been developed to forecast the SD of travel time on links and routes. A list of these models is presented below. Readers can refer to Appendix A1 which presents a detailed discussion on each of models listed below.

1. United Kingdom Model (UKM)
2. Log-Linear Model (LLM)
3. New Zealand Model (NZM)
4. Unified Reliability Model (URM)
5. Linear Model (LM)
6. Length Standardised Linear Model (LSLM)
7. Length Standardised Cubic Model (LSCM)
8. Exponential Coefficient of Variation Model (ECVM)
9. Power Mean Delay Model (PMDM-1)
10. Polynomial Mean Delay Model (PMDM-2)
11. Dutch Model (DM).

Most of these models have been calibrated as route models, that is the travel time SD of a series of interconnected links instead of individual links. However, a few studies have indicated that link-level modelling is a better paradigm since it could take into consideration the dependencies among links. For example, route travel time represents the summation of its constituent link travel times. However, some links are correlated to one another, which cannot be identified from a single number representing route travel time.

2.2.1 Comparison of Existing Link Models

The criteria used to compare the models mentioned above is as follows:

1. Consideration of the dependent variable in each model
2. Review the influence of factors such as congestion index, link length, road types and limits of variability
3. Conduct a standard numerical experiment using each of the models.

The three criteria are discussed below.

1. Consideration of the dependent variable in each model

Table 2-2 presents a comparison of the models reviewed summarising the dependent and independent variables along with the upper and lower bounds of the former. The models have generally used either SD or Coefficient of Variation (CoV) as the dependent variable.

While SD corresponds to the spread of travel time around the mean value, CoV is a normalised measure which is equal to the ratio of SD and the mean travel time on a link. The data collected for this study had link lengths ranging from below 100 m to over 10 kms. As link lengths vary across the network, an approach favouring a CoV might be preferred. The table also shows a majority of models leading to unbounded results, which implies that the estimated values of the dependent variable would be infeasible in reality.

Table 2-2: Variables and limits of the eleven existing models to estimate travel time variability

S. No.	Model	Dependent variable	Independent variable(s) included	Limits of dependent variable
1	UKM	CoV	• Congestion index specified as the ratio of mean and free flow travel time • Length of travel	• Minimum = a • Maximum = ∞
2	LLM	CoV	• Congestion index specified as the ratio of mean and free flow travel time • Length of travel • Road type assigns different parameters	• Minimum = a • Maximum = ∞
3	NZM	SD	• Congestion index specified as the ratio of volume (in terms of demand) and capacity • Road type assigns different parameters	• Minimum = σ_0 • Maximum = σ_1
4	URM	SD	• Travel time • Length of travel	• Minimum = a function of capacity, time-of-day and type of route • Maximum = ∞
5	LM	SD	• Travel time	• Minimum = a • Maximum = ∞
6	LSLM	SD per unit length	• Unit travel time (that is inverse of speed)	• Minimum = a • Maximum = ∞
7	LSCM	SD per unit length	• Unit travel time (that is inverse of speed)	• Minimum = a • Maximum = peaks at a defined unit travel time then declines to negative values
8	ECVM	CoV	• Congestion index specified as the ratio of mean and free flow travel time	• Minimum = exp(a) • Maximum = peaks at a defined congestion index then declines to zero
9	PMDM-1	SD	• Congestion index specified as mean delay (that is difference between travel time and free-flow travel time	• Minimum = a • Maximum = ∞
10	PMDM-2	SD	• Congestion index specified as mean delay • Length of travel	• Minimum = a • Maximum = ∞
11	DM	SD	• Congestion index specified as mean delay • Length of travel	• Minimum = a and increases linearly with length • Maximum = ∞

Note: a, σ_0 and σ_1 are the calibration parameters which define the boundaries of the dependent variable

2. Review of Influencing Factors

The following independent variables from the literature have been reviewed with comments whether they will be the adopted variables for comparison in this report:

1. **V/C ratio:** Volume is based on the measurement of vehicles 'arriving' from the analysis point. Volume from field data can only measure vehicles discharging or flow, but it cannot indicate upstream demand. Thus, the V/C approach in this section will use demand on arrivals rather than flow on departure. This enables V/C ratios greater than one to be tested on the 11 selected models.

2. **Congestion Index (CI):** The CI used for analysis in this report denotes a ratio of mean and free flow travel time
3. **Mean Delay:** Mean delay is the difference between expected and free flow travel time. However, mean delay is length dependent, hence the use of mean delay does not differentiate between delays due to heavy congestion on a short link and moderate delays on a long link. Hence the application of mean delay as the CI has disadvantages.
4. **Link Length:** A link is usually assumed to be homogenous, implying uniform CoV across the entire link. However, this assumption is not suitable as link length increases because various segments of the link behave in a less correlated manner, which tends to reduce the variability of travel time. Hence, longer link lengths tend to depict lower travel time variability. This report compares the models over a range of link lengths.
5. **Road Types:** Different road types exhibit different characteristics on how the congestion index translates into travel time variability. Higher standard highways, particularly motorways, tend to exhibit less travel time variability compared to lower standard highways (e.g. arterials) during off peaks since greater road capacity enables travel at the posted speed limit. However, during AM and PM peaks, arterials are often heavily congested which contribute to a large travel time variability. Different model parameters are often determined for each road type.
6. **Limits of Variability:** While all the reviewed models have defined lower bounds, it is only the NZM which has an upper bound to the travel time variability. By limiting the SD, the CoV would decline at higher congestion levels as travel time increases, while the SD stays constant beyond a certain point in the NZM. It is beneficial to constrain the models such that travel time variability does not yield unrealistically high values. However, it is more appropriate to constrain the CoV instead of the SD of travel time so that variability could be accounted for highly saturated long links.

CI can be specified as a ratio of mean and free-flow travel time; mean delay or, as a ratio of volume (in terms of demand) and capacity. Variability of travel time increases with CI. Calibration based on volume is problematic because demand is not easily measurable. Volume data is measured using traffic counters which measure the realised flow. In undersaturated conditions, the realised flow is the same as demand. However, in saturated conditions, the realised flow is much less than demand due to effect of bottlenecks constraining flow. Travel time is measured either by probe vehicles, roadside detectors, or Bluetooth tagging. The measured and modelled travel times are consistent in both undersaturated and saturated conditions, hence the CI approach using the ratio of mean and free-flow travel time have advantage over the volume-based approach and is used hereafter throughout the report.

3. Numerical experiment of each model

A numerical experiment was also developed to compare the models reviewed above. Of these models, UKM and NZM had two variants each, that is UKM – UK version and UKM – Australian version and NZM – New Zealand arterial version and NZM – Sydney arterial version. Thus, 13 models were eventually compared in the numerical experiment [4].

The numerical experiment represented a hypothetical scenario with the following characteristics:

- Arterial highway route comprising 1 link
- Speed limit: 80 km/h
- Link length: 1 km
- Capacity: 1,200 passenger car equivalent units (pceu) per lane-hr
- Travel speed and CI (ratio of mean to free flow travel time) are estimated, using the Bureau of Public Roads model (BPR, 1964).

[4] It is noteworthy to mention that the UKM and LLM had a similar function form, i.e. one can deduce LLM from UKM by taking the logarithm on both sides. As a result, the two models are expected to produce identical results in the numerical experiment. Nonetheless, both the models are still considered in this exercise as they have been used in previous studies.

The findings from the numerical experiments are summarised below [5]:

- The predicted SD of travel time for the LM and DM for a V/C less than 1 is inconsistent with other models and common understanding
- NZM predicts a constant SD of travel time for higher V/C, LSCM and ECVM forecast decreasing values with increasing V/C, and other models reported increase in SD values with increasing V/C ratio
- A majority of models predict decreasing CoV values as V/C increased
- DM and LSCM forecasted negative CoV values for a certain range of V/C
- UKM, URM and LLM displayed higher than anticipated rate of increase for CoV and SD of travel time for V/C values above 0.8.

In summary, the literature review and the numerical experiment indicate that using CoV as the dependent variable provides a better model fit when compared to SD forecasting models. The estimated CoV is then converted back to SD to measure the monetary cost of travel time reliability (Equation 1.1) to be used during the CBA. Furthermore, the literature review shows that some link models can forecast ever increasing values of CoV as the V/C or the CI increases (refer to Table 2-2). In other words, SD of travel time continues to increase as roads become more congested. This mathematical aspect, however, is not consistent with the real-world traffic phenomenon observed in general where travel time variability tends to change at a much lower rate beyond a certain threshold. It is worth noting at this point that this rate of change in the SD of travel time can be positive (that is increasing) or zero (that is constant). Thus, both potential options need to be explored using two types of trendlines: 1) where the rate of increase in SD of travel time is gradual (that is no cubic or exponential forms), and 2) where the SD of travel time stabilises and takes the shape of a plateau beyond the threshold value. The findings from the numerical experiment augment some of these observations from the literature review and the real-world traffic flow phenomenon.

Table 2-3 summarises each of the eleven models based on the findings from the literature review and the numerical experiment and identifies the models which were found appropriate for further consideration.

Table 2-3: Summary of the link models considered

Model	Review	Recommendation
UKM	• For values of b (the exponent parameter) greater than 1, the model forecasts unrealistic growth in SD of travel time • Does not have a limiting maximum threshold for forecasted CoV, thus susceptible towards over-estimating travel time reliability benefits • However, the hypothesis of geometrically increasing CoV could be tested on field data, thus shortlisted this model for further consideration.	**For further consideration**
LLM	• Forecasts an exponential increase in SD of travel time at higher congestion which is unrealistic • Does not have a limiting maximum threshold for forecasted CoV, thus susceptible towards over-estimating travel time reliability benefits.	Not for further consideration
NZM	• Predicts a sigmoid trend in SD of travel time as congestion increases, that is SD of travel time stabilises beyond a threshold value • The hypothesis of a constant SD of travel time at higher congestion could be tested on field data, thus shortlisted this model for further consideration.	**For further consideration**
URM	• Travel time (independent variable) is not a good indicator for congestion. E.g. change in speed limit could also impact travel time • Does not have a limiting maximum threshold for forecasted SD of travel time, thus susceptible towards over-estimating travel time reliability benefits at high congestion levels.	Not for further consideration

[5] Readers can refer to Appendix A2, Section 1 which presents the results of the numerical experiment.

Model	Review	Recommendation
LM	• Travel time (independent variable) is not a good indicator for congestion. E.g. change in speed limit could also impact travel time • Its simple form does not account for the increasing rate of SD of travel time as the link approaches saturation levels, which is unrealistic • Tendency to over-estimate travel time variability at lower and underestimate at higher congestion levels.	Not for further consideration
LSLM	• Uses travel time as independent variable which is not a good indicator of congestion • Limited flexibility to model the transition from lower to higher congestion.	Not for further consideration
LSCM	• Uses travel time as independent variable which is not a good indicator of congestion • The cubic exponent leads to unrealistic drop in SD of travel time at higher congestion.	Not for further consideration
ECVM	• Forecasts a zero CoV at higher congestion level, which is unrealistic.	Not for further consideration
PMDM-1	• Uses mean delay as independent variable is not suitable as it is length dependent • Thus, unable to distinguish between delays due to heavy congestion on a short link and moderate delays on a long link • The logarithmic curve (that is the exponent parameter (b) less than one) gives a realistic estimate of SD of travel time • For larger values of b, the model forecasts exponential growth in SD of travel time which is unrealistic.	Not for further consideration
PMDM-2	• Uses mean delay as independent variable is not suitable as it is length dependent • The cubic exponent is susceptible for unrealistic changes in SD of travel time.	Not for further consideration
DM	• Functional form is weighted sum of linear trend and a log trend • The CoV could decrease at higher levels of congestion • The problem of predicting negative values of SD of travel time could be addressed by constraining the parameters to prevent it from returning negative values of SD of travel time • The model predicts declining CoV at higher congestion levels.	**For further consideration**

2.2.2 Selected Models

The above exercise (literature review and numerical experiment) brought forward three link level models for further consideration namely, UKM, NZM and DM. Table 2-4 summarises the shortlisted models along with the justification behind their selection. It was proposed that these models would be compared with real-world data to determine the best fitting model.

Table 2-4: Recommended three link models for analysis

Model	Predicted Modelled Behavior
UKM	Geometrically increasing CoV at higher levels of congestion.
NZM	Constant SD of travel time and sharply declining CoV at higher levels of congestion.
DM	The DM predicts that the CoV would decline at higher levels of congestion. The DM predicts a negative CoV value at certain CI ranges which implies its parameters would need to be constrained.

2.3 Route Travel Time Variability

In travel time reliability modelling, all related parameters are link based, which are then aggregated to measure route performance. The impact of motorway on/off ramps, traffic signals, junctions and roundabouts will be considered at link level, that is, travel time and SD, and then reflected in the overall route performance.

Regarding the modelling of route performance for CBA, the mean route travel time can be calculated as the sum of link travel times along the route. The SD of route travel time can only be aggregated from the link travel time SD, if the link travel times are uncorrelated (or independent). As some sources of variability, e.g. weather or high demand, impacts upon a wide section of links in the network, it is reasonable to assume that there is some level of correlation between link travel times that is if a link is experiencing congestion it is likely that nearby links are also experiencing congestion. The review identified two approaches to modelling route travel times, that is travel time-based models such as the UKM Route Model and Correlation Route Model (CRM).

2.3.1 Travel Time-based Route Models

Moylan et al. (2018) applied the UKM model to a route and expressed the UKM as shown in Equation 2.2 along with Table 2-5 which gives the associated calibrated parameters.

$$\boldsymbol{\sigma = at^b d^c} \quad \text{[EQ 2.2]}$$

Where:

σ = SD of travel time of route (in s)

t = mean travel time of route (in s)

d = distance of route (in km)

$\mathrm{a, b\ c}$ = parameters

UKM calibrates based on mean travel time, which is not a measure of congestion. Information about individual links such as the congestion index is lost because of the aggregation. Therefore, the application of the UKM to a route is not sensitive to congestion and it is suitable only for approximating the standard deviation of route travel time. The model is accurate only for the route used in its calibration and limited to the configuration of the route at the time of calibration.

Table 2-5: Calibrated parameters of UKM

Source	Road Stereotype		a (Constant)	b (Travel time)	c (Length)
UK Department for Transport's Transport Analysis Guidance, Moylan et al. (2018)	General		0.0018	2.02	-1.41
Sydney Highways (rCITI 2018)	General (arterials & motorways)		0.0087	1.5245	-0.4488
	General by time-of-day	AM peak	0.0082	1.6349	-0.5926
		PM peak	0.7256*	0.7187	0.1626
		Interpeak	0.7245*	0.7144	-0.0098*
		Off-peak	0.0139	1.3166	-0.2129
	Arterial	General	0.0001	2.4917	-1.5383
	Arterial by type of route	Inner ring	1.97607×10^{-7}	3.4058	-2.7373
		Middle ring	9.92653×10^{-6}	3.1107	-2.2848
		Outer ring	4.33235×10^{-6}	2.7885	-1.0203
	Arterial by time-of-day	AM peak	0.0003	2.3857	-1.4027
		PM peak	0.0166	1.5081	-0.6501
		Interpeak	0.0419	1.3667	-0.7839
		Off-peak	0.0002	2.3043	-1.3974
	Motorway	General	1.03251×10^{-7}	4.2451	-2.7198
	Motorway by type of route	Inner ring	8.82934×10^{-8}	4.4697	-3.3849
		Middle ring	2.99007×10^{-7}	3.6758	-1.4987

Source	Road Stereotype		a (Constant)	b (Travel time)	c (Length)
		Outer ring	6.83866×10^{-14}	7.8569	-6.0742
	Motorway by time-of-day	AM peak	1.35447×10^{-6}	3.6946	-2.3010
		PM peak	0.0002	2.5966	-1.3939
		Interpeak	1.73597×10^{-9}	5.3052	-3.7987
		Off-peak	4.05581×10^{-10}	5.4423	-3.6477

*Note: * indicates that the parameter was not statistically significant.*

2.3.2 SD of Travel Time Based Route Model

The SD of travel time based route model involves forecasting the link travel time SD (using the models discussed in Section 2.2) followed by combining the link travel time SDs to form the travel time SD for a route. This method has been widely used in the previous works reviewed earlier in Section 2.2. There are two approaches for SD of travel time-based route models, either assuming that there is (i) a correlation between links or that (ii) the travel time variability of each link is uncorrelated.

The first approach, shown in Equation 2.3, referred to as the Correlation Route Model (CRM), was recommended by Nicholson (2015) to include consideration of travel time correlation between all links within a route. Variance of route travel time is defined as the sum of variance of link travel time and the sum of covariance between any two links' travel time. This is in fact the most accurate measure of variance between datasets by using the variance sum law from statistical theory. However, this model increases the level of complexity in the variance calculation and it is heavily reliant on available data to determine the correlation coefficient of travel time between any two links. Nicholson (2015) also developed Equation 2.5 to estimate the correlation coefficient in order to simplify the calculation.

$$\boldsymbol{\sigma_r^2 = \sum_{i=1}^{n} \sigma_i^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \rho_{i,j}\sigma_i\sigma_j}\,, i < j \qquad \text{[EQ 2.3]}$$

Where:

σ_r^2 = variance of travel time of route with n number of links

σ_r = SD of travel time of route

σ_i^2 = variance of travel time of link i

σ_i = SD of travel time of link i

σ_j = SD of travel time of link j

$\rho_{i,j}$ = correlation coefficient of travel time between links i and j

The second term of Equation 2.3 consists of $\frac{n^2}{2} - n$ terms, the upper triangular part of an $n \times n$ matrix excluding the *n* terms along the diagonal.

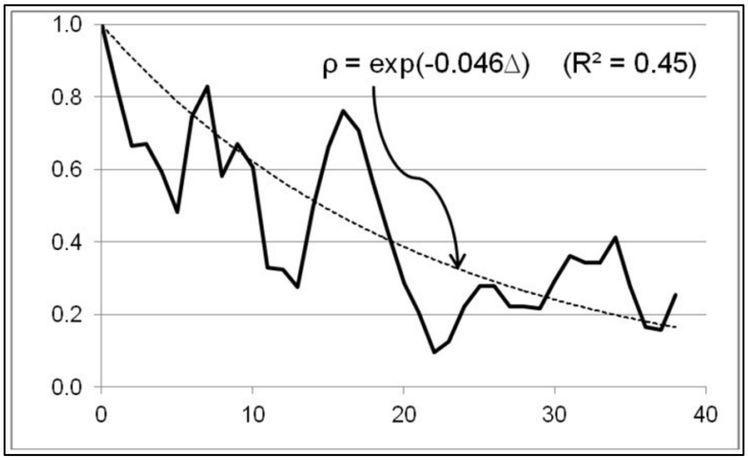
The second approach, as shown in Equation 2.4 assumes that the estimate of trip reliability for a journey could be built up using individual variability on links and junctions (measured in terms of variance) and that there is no correlation effect in the variance between the links and junctions (Osterle et al., 2017). The NZTA Economic Evaluation Manual also assumes that travel times are independent, thereby the correlation coefficient is assumed to be zero (NZTA, 2013). If the correlation coefficient is zero, then Equation 2.3 simplifies to Equation 2.4.

$$\boldsymbol{\sigma_r^2 = \sum_{i=1}^{n} \sigma_i^2} \qquad \text{[EQ 2.4]}$$

Moylan et al. (2018) determined that expressing the SD of route travel time as Equation 2.4 would result in under-estimating SD of route travel time by 43.5% with their field data. Nicholson (2015) using field data estimated that the contribution of the variance term in (that is first term in Equation 2.3) is around 9% or the covariance term is roughly 10 times greater (that is second term in Equation 2.3). Therefore, link travel times are not independent, and correlation needs to be accounted for when calculating the SD of route travel time.

Nicholson (2015) used field data of Tokyo Metropolitan Expressway to identify the relationship between links travel time and found that the correlation coefficient of link travel time between any two links varies but averages around 0.5. He noted that the correlation coefficient tended to decline with increasing separation between links as shown in Figure 2-2.

Figure 2-2: Correlation coefficient (ρ) of link travel times versus link separation (Δ)



Note: The correlation coefficient is between first link on the route with the next 38 downstream links along the route. Separation (Δ) is the number of links from the first link on the route. The links are 300 ± 50 m in length.
Source: Nicholson (2015)

Nicholson (2015) proposed Equation 2.5 to estimate the correlation coefficient. Separation in Equation 2.5 is the number of links between two links along a route wherein link lengths are approximately uniform that is, 300 ± 50 m.

$$\boldsymbol{\rho_{i,j}} = \boldsymbol{exp}(\boldsymbol{a}\Delta) \quad \text{[EQ 2.5]}$$

Where:

$\rho_{i,j}$ = correlation coefficient of travel time between links i and j

Δ= separation, number of links (link lengths ≈ 300m)

a = parameter

The use of the CRM is considered more accurate, hence is a more appropriate method of estimating route travel time. Travel time-based route models do not have the necessary generality and sensitivity to be adopted in a guideline. The CRM can take advantage of link-based models to aggregate the route travel time SD of travel time. The only complication with the CRM approach is the need to calibrate a set of correlation coefficient models (CCM, that is Equation 2.5). Preliminary analysis has demonstrated that it is necessary to account for the correlation of links. With a set of calibrated parameters for the CCM, the application of the CRM (that is Equation 2.3) is straightforward and would result in higher quality estimates.

It is worth noting that the correlation coefficient model relies heavily on calibration using field data and results can be significantly different depending on the road environment. It is essential to test the model to determine whether it can be generalised for different networks or regions.

2.4 Travel Time Variability in Network Modelling

Network models that measure road network performance are tools used by practitioners to develop strategic infrastructure plans and prioritise transport investment. Accordingly, considerable efforts have been made to enhance the realism of these network models, especially in the context of improving traffic assignment methodologies. The traffic assignment process concerns the allocation of travel demand on feasible routes for each origin-destination pair within a road network model. This allocation is governed by factors which affect travellers' route choice such as travel time, cost as well as travel time reliability.

There is an extensive body of literature concerning the incorporation of travel time reliability within network modelling approaches. Taylor (2013) and Uchida (2014) provide comprehensive documentation of the research history while recent papers by Gupta et al. (2018) and Mishra et al. (2018) discuss the application of travel time reliability within the transport planning appraisal process.

Current models incorporate travel time reliability within network models in two ways:

- traffic assignment methodologies which can measure reliability
- traffic assignment methodologies that include travel time reliability metrics within the route choice process (Taylor, 2013, Sun et al., 2019).

2.4.1 Review of Existing Approaches

Historically, the network models focused exclusively on reliability measurement, such as Asakura and Kashiwadani (1991) which solved the User Equilibrium for numerous origin-destination (O-D) demands sampled from a normal distribution to estimate the SD of travel time for links within a network. However, with the development of assignment methodologies such as Stochastic User Equilibrium (SUE) (Prashker and Bekhor, 2004), Strategic User Equilibrium (StrUE) (Dixit et al., 2013) and a plethora of dynamic traffic assignment techniques that integrate uncertainty within the assignment process itself, models are able to both measure travel time reliability and incorporate it within the assignment of traffic. For example, Strategic User Equilibrium (StrUE) probabilistically assigns traffic to the network as it accounts for the variability in O-D demands which result in variations of travel time. The modelling approach can then measure reliability directly from the modelled link proportions as the variance of link travel time.

Sun et al. (2019) provides a categorisation of route choice equilibrium-based traffic assignment models which account for travel time reliability summarised in Table 2-6:. Travel time reliability models relevant to the metrics identified earlier in the report focus on the 'mean-variance' approach where link cost functions or route decision rules consider either the variance, SD of travel time or percentile of travel time as a component. This mean-variance approach will be pursued in more detail within the models developed in the project.

Table 2-6: Categorisation of network models which incorporate travel time reliability

Type of model	Description	Examples
Mean-variance approach	These traffic assignment models include either expected travel time, travel time variance or SD of travel time to estimate the link/route travel cost. Travelers choose routes to minimise this travel cost (to avoid late arrival) – which could be interpreted as a travel time budget. These models may account for uncertainties in supply, demand or both.	• Lo et al. (2006) (Travel time Budget – TTB Model) • Wu (2015) (TTB Model) • Shao et al. (2006) (TTB Model) • Nie (2011) (Percentile Travel Time Model) • Chen et al. (2011) (Mean excess traffic equilibrium) • Dixit et al. (2013) (StrUE) • Clark and Watling (2005) (SUE)
Game-theoretic	These models assume that travelers choose routes to avoid link failure (avoid unreliability) and that disruptors maximise the damage to the network. The traffic assignment methodology is then formulated as a Cournot-Nash game.	• Szeto et al. (2007)

Type of model	Description	Examples
Prospect-theory based	Travelers choose the route with the largest prospect value to complete a journey. Travelers are risk averse in positive (gain) scenarios and risk prone in negative (loss) scenarios.	• Gao et al. (2010) • Chorus (2012) • Li et al. (2016)

Recent research by Gupta et al. (2018) and Mishra et al. (2018) present the integration of travel time reliability in regional travel models with practical case studies. Gupta et al. (2018) presents a study based on Phoenix, Arizona. The key methodological contribution surrounds the manipulation of speed and volume data to develop a reliability measure that guided path generation in a traffic assignment process. This was extended to define O-D reliability measures constructed from the link-level and route-level reliability measures, including the SD of travel time. The paper confirms recent literature from Moylan et al. (2018) that SD of travel time is not additive, and it is important to account for correlations between links along a route.

Similarly, Mishra et al. (2018) also develops an OD based reliability measure which was integrated with the Maryland State-wide Transportation model to find the value of reliability savings by improving road links within the network. The paper proposed a methodology to measure the value, forecast and incorporate reliability in the transport planning process and can serve as a possible template for the work undertaken in this research study.

3 Methodology

This section presents the methodology adopted for further consideration based on the findings from the literature review section. Section 3.1 discusses the methodology adopted for link travel time variability and is compared with the models shortlisted in sub-section 2.2.2. Section 3.2 presents the modelling approach to determine route travel time reliability, which also involves the link model. Section 3.3 presents two methods to model network level travel time variability and discusses the merits and demerits for each method.

A link is the fundamental building block in a road network which is subjected to day-to-day travel time fluctuations. Thus, sub-section 3.1 emphasises more on the link-level travel time variability modelling which is then contributes to the calculation of more macroscopic elements. For example, the CRM adds the travel time variability of individual links forming a route along with the covariances which exist due to interdependencies among links.

3.1 Modelling Link Travel Time Variability

As discussed earlier in Section 2, the three shortlisted models, UKM, NZM and DM, were found to have some shortcomings (refer to sub-section 2.2.1). Thus, a fourth model, referred to as the ATAP link Model was formulated and compared with the other three models. The ATAP link model addressed some of the limitations associated with the other models such as:

- It has a ceiling for the maximum CoV value, where UKM does not. The lack of a constraining maximum coefficient in the model structure limits the UKM in modelling the sharp increases in CoV at near saturation and at the same time reduces the over-estimation SD of travel time at high levels of congestion.
- It leads to a gradual increase in CoV value rather than a decreasing CoV as the congestion index value increases. The NZM predicts a nearly constant SD of travel time at much higher levels of congestion. The NZM is unlikely to produce travel time reliability benefits for any interventions at high congestion levels.
- The use of mean delay as its measure of congestion in the DM was problematic as the SD of travel time was length dependent. The DM could not differentiate between delays due to heavy congestion on a short link and moderate delays on a long link. Its functional form could not accurately predict SD of travel time values at low congestion levels and results in an unrealistically low (or even negative) value.

3.1.1 ATAP Link Model

Equation 3.1 gives the formula for the ATAP link model.

$$\boldsymbol{CoV = a\left[\frac{(CI-1)}{CI}\right]^{b} \;\forall\; CI \geq 1} \qquad \text{[EQ 3.1]}$$

$$\boldsymbol{Ln(CoV) = Ln(a) + b.Ln\left(\frac{CI-1}{CI}\right) \;\forall\; CI \geq 1} \qquad \text{[EQ 3.2]}$$

Where:

CoV = coefficient of variation, $\frac{\sigma}{T}$

CI = congestion index $= max\ (1, \frac{T}{T_f})$, where $T = mean\ travel\ time, T_f = free\ flow\ travel\ time$

a = calibration parameter that sets the upper limit of CoV, $\mathrm{a} \mid \mathrm{a} > 0$

b = calibration parameter that determines the rate at which CoV approaches the maximum, $\mathrm{b} \mid \mathrm{b} \in (0, 1)$

The mathematical formulation of the ATAP link model (Equation 3.1) resembles that of an exponential curve. As per definition, the Congestion Index, which is the ratio of prevailing and free flow travel time, ideally could not be less than one (assuming drivers do not indulge in over-speeding beyond the free-flow speed). Thus, an exponential curve helps in modelling the scenarios when the CI is greater than or equal to one, with the outliers being the scenarios where CI is less than one. The model gives a minimum CoV of zero when the CI is equal to one and then increases at a gradually diminishing rate for higher CIs. This functional shape is considered consistent with real-world traffic.

Equation 3.1 can be transformed by taking a natural logarithm on both the sides, which is shown in Equation 3.2. The benefits are that the log-log model, being linear, allows for an easy calibration using the linear regression tool available in MS-Excel (and other statistical packages), thus making it practitioner friendly. Secondly, using a log-transformed variable mimics the law of diminishing returns which implies slower rate of increase at higher variable values. For example, while a CI of 10 can be considered high, Ln[(CI-1)/CI] equates to -0.105 which greatly compresses the spread in the variables. Lastly, from a real-world standpoint, the CoV value is expected to follow the law of diminishing returns, that is the travel time variability reduces as at higher traffic congestion levels (since traffic movement is significantly slower). All these reasons favour the model calibration using a log-transformed model (Equation 3.2) as it would facilitate better model goodness-of-fit.

3.1.2 Comparing the ATAP Link Model and other Shortlisted Models

The four models, that is, UKM, NZM, DM and the ATAP link model were again developed for the same numerical experiment environment discussed earlier in sub-section 2.2.1. Readers can refer to Appendix A2, section 2 which presents the results from this numerical experiment. Some of the findings from the numerical experiment have been summarised below:

- Our study both theoretically and using field data concluded that, at the link level, travel time SD is length dependent while travel time CoV is length independent
- Numerical analysis in Appendix 2.2 compared those models and found only UKM-Australia and ATAP model matched the conditions that travel time SD is length dependent and travel time CoV is length independent
- The ATAP link model shows a reasonable curve[6] for both the CoV (a stable value at higher CI) and the SD (increasing steadily with CI)
- The ATAP link model has a good fit[7] with the trendline observed in the NetPReS data.

Based on the analysis and comparison along with the advantages listed above, it was determined that the ATAP link model was the most appropriate method of measuring variability at a link level.

The ATAP link model was calibrated and validated on the available real-world datasets, the results of which are presented in Section 4 of this report. The methodology for the calibration and validation of the ATAP link model is summarised below.

Step 1:	Obtain the CI and CoV of travel time information separately for all arterial and freeway links in the Perth based NetPReS data
Step 2:	Separately calibrate the ATAP link model on each of the arterial and the freeway data to determine the estimated parameters and the goodness-of-fit of the model. A nonlinear regression method is to be used for model calibration
Step 3:	Visually validate the two ATAP models (for arterials and freeways) using other available datasets such as the Gold Coast Bluetooth dataset and the Sydney Google dataset
Step 4:	Record findings and make recommendations

[6] A reasonable curve implies that the trendline is consistent with the traffic conditions witnessed in real-world.

[7] A good fit implies that the forecasted values (i.e. model trendline) closely follows the one formed using observed values.

3.2 Modelling Route Travel Time Variability

In order to predict the SD of travel time on a route, the CRM presented in Equation 2.3 was applied: ($\boldsymbol{\sigma_r^2} = \sum_{i=1}^{n} \boldsymbol{\sigma_i^2} + \mathbf{2}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \boldsymbol{\rho_{i,j}\sigma_i\sigma_j}\,, i < j$).

The CRM model is comprised of two sub-models, the ATAP link model and the Correlation Coefficient Model (CCM), each of which was calibrated using the entire NetPReS dataset available for analysis. It is important to note that the modelled route travel time variance, that is $\boldsymbol{\sigma_r^2}$, is estimated as a summation of the variances of the constituent links (an output from the ATAP link model) along with the total travel time variances due to correlations among the links (an output from the CCM). The sub-models were calibrated using the NetPReS link information while the validation of the resulting CRM was conducted using the NetPReS route dataset.

The validation of CRM required comparison of the modelled route travel time variance ($\boldsymbol{\sigma_r^2}$) with the measured route travel time variance which was calculated as adding the individual link travel times to determine the route travel time for a given time-period and day, followed by calculating the variance of route travel times across all working days, excluding public holidays, in a month.

In the case where the correlation coefficient (ρ) of travel time between the links is not available, Equation 2.5 ($\boldsymbol{\rho_{i,j}} = \boldsymbol{exp}(\boldsymbol{a}\Delta)$) can be used to estimate the value of the correlation coefficient.

Equation 3.3 shows an adaptation of Equation 2.5 where the number of separating links has been replaced with distance (midpoint to midpoint) between two links. This transformation is expected to yield similar results as the two quantities are positively correlated to one another. The latter quantity makes the application of the equation quite easy and computationally straightforward, which is why it has been adopted.

$$\boldsymbol{\rho_{i,j}} = \boldsymbol{exp}(\boldsymbol{aL}) \qquad \text{[EQ 3.3]}$$

Where:

$\rho_{i,j}$ = correlation coefficient of travel time between links i and j

L = distance between the midpoints of two links (in km)

a = parameter

A correlation analysis was conducted to test the validity of Equation 3.3 against the real-world data. Readers can refer to Appendix A4 which presents the results from the route correlation analysis. It was observed from the analysis that the exponential functional form given by Equation 3.3 resulted in a poor model fit[8] when applied on the Perth dataset. Thus, a new functional form was proposed to estimate the correlation coefficient ($\rho_{i,j}$) which is presented in Equation 3.4.

$$\boldsymbol{\rho_{i,j}} = \boldsymbol{Max}[\mathbf{0}, \boldsymbol{a}.\mathbf{Ln}(\mathbf{L}) + \mathbf{b}] \qquad \text{[EQ 3.4]}$$

Where:

$\rho_{i,j}$ = correlation coefficient of travel time between links i and j where i < j

L = distance between the midpoints of two links (in km)

a, b = parameters

[8] A poor model-fit implies that the exponential trendline did not quite pass through the middle of the observed datapoints. In other words, the exponential trendline looked a bit skewed on the Perth dataset.

This study focussed on the impact of link distance on the travel time correlation between the links in general. The data includes observations with negative correlation which are included in regression to fit the curve. The fitted curve forecasts a negative correlation for higher values of L. Examination of the exact causes of negative correlation was out of study scope, however the initial investigation suggested that it was more likely to be attributed to the geometrical settings, bottleneck effects or other causes rather than distance between links. It is recommended for future study to fully investigate the factors affecting travel time correlation between links and further enrich Equation 3.4 by incorporating additional independent variables into the function. Equation 3.4 has been written to exclude negative correlations, and this form should be used in practical applications (as negative correlations due to longer mid-distance does not make sense).

Once the correlation coefficient (ρ) is determined, it can then be used in Equation 2.3 to calculate the travel time variability on a route. The calibration methodology for the route level travel time variability modelling is summarised below:

Step 1:	Calibrate the CCM (Equation 3.4) by linear regression of correlation coefficients (including negative values) against the natural logarithms of distances between the midpoints of links. Summarise the calibrated parameters, statistical significance and the goodness-of-fit of the model
Step 2:	Calculate the variance of route travel time using the full NetPReS data on Equation 2.3 that is, $\sigma_r^2 = \sum_{i=1}^{n} \sigma_i^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \rho_{i,j}\sigma_i\sigma_j$, where σ_i is determined using the ATAP link model and $\rho_{i,j}$ is determined from Step 1
Step 3:	Calculate the sum of the instantaneous link travel times as route travel time and measure the day-to-day SD of route travel time, σ_r using the NetPReS data
Step 4:	To validate the proposed CRM, compare the SD of route travel time predicted from Step 2 against measured SD of route travel time from the NetPReS data
Step 5:	Record findings and make recommendations

The assumption of expressing the route travel time as the instantaneous sum of link travel time was examined by Moylan et al. (2018) and it was found to be accurate to within 4% on average and there is a tendency to overestimate route travel time particularly in congested periods and long routes. The loss of accuracy is thereby relatively small in relation to the benefit of simpler calculation.

3.3 Modelling Network Travel Time Variability

The literature review highlights several approaches to account for travel time reliability within a road network performance assessment (refer to Section 2.4). This project was built from the studies by Gupta et al. (2018), Mishra et al. (2018), Dixit et al. (2013) and Moylan et al. (2018) to test two methodological approaches, given below, that incorporate travel time reliability into strategic network modelling:

1. Inclusion of a reliability metric (SD of travel time) within the link cost function for the network accounting for the additive properties between links and routes of the network.
2. Application of the Strategic User Equilibrium (StrUE) approach which inherently accounts for SD of travel time as a variable within the assignment process.

3.3.1 Inclusion of a Reliability Metric

The first method can be readily adopted in traditional network assignment techniques (such as User Equilibrium (UE)), but would require a simplifying assumption on the estimation of route travel time SD. This approach, referred to as the Approximated Route Standard Deviation method (ARSD), assumes that the route travel time SD can be approximated by Equation 3.5. The key difference between the ARSD and the CRM (Equation 2.3) is that it excludes travel time correlation between all links within a route (which is represented by the second term on the right-hand side of Equation 2.3). While this approximation is not sufficient for estimating route travel time SD, it will greatly simplify the calculation involved in determining network equilibrium. The route travel time SD can be estimated using more accurate models post route assignment (that is Equation 2.3, Equation 3.1, and Equation 3.4).

The ARSD approach is a new approach which was proposed for consideration in this project. Readers can refer to Section 3 of Appendix A2 to get an understanding of the magnitudes of the correction factor (γ) as the correlations among links and their lengths vary.

$$\boldsymbol{\sigma_r \approx \gamma \sum \sigma_l} \qquad \text{[EQ 3.5]}$$

Where:

$\sigma_r =$ SD of travel time on route

$\sigma_l =$ SD of travel time on links

$\gamma =$ correction factor

3.3.2 Proposed Network Modelling Application using StrUE

In a network modelling context, travel time reliability can be incorporated exogenously by measuring the consequent travel time variability over several scenarios, or endogenously by accounting for travel time reliability within the traffic assignment methodology. To realistically capture the concept of travel time reliability within a transport system, it is important to incorporate the value of reliability within a traveller’s decision-making process to evaluate the impact on route choice and the emergent expected travel times and variability. Accordingly, endogenous approaches[9] to modelling travel time reliability within strategic frameworks provide the greatest realism when accounting for travel time reliability because it implicitly accounts for travel time reliability because of uncertainty in demand or capacity and its impact on route choice. At its core, the main cause for unreliability is due to stochasticity in the demand or capacity[10]. StrUE explicitly accounts for this stochasticity in demand and capacity, while the behavioural model accounts for the preferences over travel time and reliability to evaluate the network impact.

An approach that has been tested within this project is the StrUE traffic assignment formulation (Dixit et al., 2013). StrUE is a novel formulation that accounts for the variability that exists in road networks while still maintaining the beneficial properties such as consistency and convergence of traditional traffic equilibrium models. StrUE considers that travellers recognise the variability in the system, in terms of road capacity, demand and travel time and rationally chooses routes while weighting the expected travel time and its variance. A summary of the formulation of StrUE is presented below:

StrUE is defined such that *"at Strategic User Equilibrium all used paths have equal and minimal generalised cost over expectation and variability of network demand".*

StrUE assignment relies on the following user behavioural assumptions:

---

[9] Endogenous approaches imply a two-way interaction between travel time reliability on a route and users route choice behaviour on day-to-day basis which is solved iteratively until StrUE is attained.

[10] Stochasticity in demand implies day-to-day variation in the number of users in the network. E.g. weekday versus weekend traffic. Similarly, stochasticity in capacity implies inter-day variation in the available road network supply. E.g. normal road conditions versus work zones.

1. There is a known probability function for network demand.
2. Each user will select the minimum generalised cost path (over expectation and variability).
3. Each user will follow the minimum generalised cost path under each demand realisation where that user is present.

It is important to recognise that under StrUE conditions, the path (and link) proportions will not change day-to-day. However, the actual link flow volumes will vary as a function of the realised demand (since the link flows would be the product of the realised demand and the link proportions), meaning equilibrium conditions are unlikely to be met for each independent demand realisation. This outcome is consistent with real-world road networks where equilibrium conditions are not observed on a day-to-day basis. One of the strengths of this approach is that the uncertainty in travel times and flows can be analytically tied back to demand uncertainty.

Note, the major consequence of assumption #3 is that, for each origin-destination, path flow proportion will be equal under all demand scenarios. This is because each user is represented as an infinitesimal unit of flow, which is common for continuous network equilibrium formulations. Therefore, each path will be altered proportionally when the total origin-destination demand varies.

To compute link flows, the strategic model equilibrates based on expected conditions as opposed to deterministic costs. In addition, the link travel time variability can be shown to be strictly a manifestation of travel demand uncertainty, which highlights a significant mathematical advantage of the StrUE model. The mathematical formulation for the problem is presented below. The notation used in the guidelines are provided in Table 3-1.

Table 3-1: Notations used for the StrUE model

Symbol	Meaning
A	Link (index) set
v_t	Value of time
v_r	Value of travel-time reliability
f_a	Proportion of total demand on arc a; **f**=$(\ldots,f_a,\ldots)\ \forall a \in A$
t_a	Travel time on arc a; **t**= $(\ldots,t_a,\ldots)\ \forall a \in A$
p_k^{rs}	Proportion of flow on path k, connecting OD pair r-s; $\mathbf{p^{rs}}$= $(\ldots,p_k^{rs}, \ldots)$; **p**=$(\ldots,p^{rs},\ldots)$
c_k^{rs}	Travel time on path k connecting OD pair r-s; $\mathbf{c^{rs}}$= $(\ldots,c_k^{rs}, \ldots)$; **c**=$(\ldots,c^{rs},\ldots)$
q_{rs}	Fraction of total trips that are between OD pair r-s; $1 = \sum_{\forall rs} q_{rs}$
T	Random variable for total trips with probability distribution $g(T)$
g	Probability distribution for of the total trips T
$\delta_{a,k}^{rs}$	Link-Path indicator variable $\delta_{a,k}^{rs} = \begin{cases} 1 & \text{if link } a \text{ is on path } k \text{ between OD pair } r-s \\ 0 & \text{otherwise} \end{cases}$ $(\Delta^{rs})_{a,k} = \delta_{a,k}^{rs}$; Δ= $(\ldots, \Delta^{rs}, \ldots)$
w	Variable that indicates the proportion of total trips considered in the integration of the objective function.

As path flow proportions will be fixed, link flow proportions will be as well. Therefore, the developed StrUE formulation finds the proportion of flow on each link such that all used paths between an OD pair have the same expected travel time:

$$\min z(f) = v_t \int_0^\infty \sum_a \int_0^{f_a} t_a(wT)\, g(T) dw dT + v_r \left(\int_0^\infty \sum_a \int_0^{f_a} t_a^2(wT)\, g(T) dw dT - \sum_a \int_0^{f_a} \left(\int_0^\infty t_a(wT) g(T) dT \right)^2 dw \right) \quad [3.6]$$

Subject to:

$$\sum_k p_k^{rs} = q_{rs} \qquad \forall r, s \qquad [3.7]$$

$$p_k^{rs} \geq 0 \qquad \forall r, s \qquad [3.8]$$

$$f_a = \sum_r \sum_s \sum_k p_k^{rs} \delta_{a,k}^{rs} \qquad \forall r, s \qquad [3.9]$$

In this formulation, the objective function is the sum of the integrals of the expected value of the link performance functions. Equation 3.7 represents a set of flow conservation constraints, that is the sum of the proportions of total demand on all paths *k* connecting origin *r* to destination *s* should be equal to the fraction of total network demand travelling between OD pair *r-s*. Equation 3.8 ensures the proportions p_k^{rs} are non-negative. The network structure enters the formulation through Equation 3.9, which defines the link proportion of the total demand in terms of path proportions.

In essence, a traveller develops a route choice strategy based on their preference for travel time and reliability, which they follow regardless of the realised travel demand/capacity on any given day. Accordingly, StrUE equilibrates on the "expected conditions", where links have a mean and SD of travel time and flow (follow a demand distribution, as opposed to the traditional deterministic approach). As a result, it is possible to analytically characterise the variance of travel time (a measure of reliability) on a route which then feeds into the cost function used to determine route assignment at equilibrium conditions. Therefore, travel time reliability is incorporated within the route choice component of the assignment and also can be measured as an output from the model.

The StrUE traffic assignment approach could theoretically replace the final step of traditional 4-step strategic travel models such as the Sydney Travel Model (STM) and is a promising modification to capture the concept of travel time reliability. This project includes an application of StrUE on the Sydney road network to determine travel time SD on a few selected routes (refer to Section 3 of Appendix A3 for more information on the Sydney dataset). The application methodology used can be summarised as follows:

- StrUE requires standard network geometry inputs of the study area as well as the proportion of OD demands to initiate the model. An assumption will be made on the distribution of the total network demand based on literature and available data sources. The total demand realisation varies according to the defined probability distribution where link and routes are perfectly correlated. This assumption ensures a closed form formulation as is a reflection that over time peoples route choice and behaviour becomes consistent.
- The mean and SD of link travel times are required to calibrate and validate the model. The SD of the total demand for the network is the key calibration parameter. The StrUE traffic assignment is executed by assuming a value for the SD of travel time of total network demand and the resulting output link travel times and SD of travel times will be compared against the collected data. The value of the SD of travel time of total network demand will be adjusted until the modelled travel times equal the collected travel time data.

Like other strategic models, the key traffic performance results such as link travel times and link volumes can be extracted in an application of the model. As stated above, the SD of link travel times across the network is a direct output of the model providing an indication of the spatial distribution of travel time variability as a measure of travel time reliability.

3.3.3 Proposed Network Modelling with ARSD and StrUE

The application of transport models is dependent on the purpose of the transport model. For example, understanding the impact of minor modifications to land use or signal timing may only have localised network impacts requiring the application of a simple model to assess travel time reliability performance at an approximate level, such as the proposed ARSD. However, in order to study the changes in travel time reliability due to significant infrastructure changes (e.g. introduction of a new development or construction of a new link), it is essential to capture the correlations between infrastructure and behaviour throughout the network as these changes have non-linear impacts across the network. These situations require complex network models such as the proposed StrUE model. As there are fundamental differences in the modelling technique (ARSD captures correlations between link travel times exogenously while StrUE captures it endogenously accounting for reliability of each link and route in users route choice), it is critical to evaluate and present both options.

Both models are evaluated and compared so that practitioners have multiple options when trying to appraise travel time reliability at a network level. Accordingly, the methodology is as follows:

- Demonstrate the differences between the standard static network modelling approach with those of the ARSD and StrUE models
- Develop ARSD and StrUE on the Sydney road network and using Google data
- Conduct sensitivity analysis of demands to understand the costs and benefits of each approach.

4 Calibrated Models

This section presents the calibration and validation results for the day-to-day link travel time variability model (Section 4.1), the day-to-day route travel time variability model (Section 4.2) and the day-to-day network travel time variability model (Section 4.3) by applying the proposed methodology (discussed earlier in Section 3) on the available datasets.

4.1 Link Travel Time Variability Model

4.1.1 Calibration of ATAP Link Model

The ATAP link model, shown in Equation 3.1, that is $\boldsymbol{CoV} = a\left[\frac{(CI-1)}{CI}\right]^{b} \forall\, CI \geq 1$, was calibrated using the full NetPReS dataset that was available for analysis (as discussed in Appendix A3). The raw data comprised 15-minute travel speed information, along with link lengths, for all links across a four-month period, that is August to November 2018. The daily data was available between 5am and 9pm which equates to 64 15-minute time intervals (refer to Appendix A3 for details of the NetPReS data). The dataset was divided into two groups: (i) arterials and (ii) freeways.

The ATAP link model was developed for each subset using the log-transformed model shown in Equation 3.2. The estimated dependent variable Ln(CoV) was then converted back into CoV, which represents Equation 3.1. Before model calibration, the dataset was prepared by following a filtering and processing procedure. The filters were applied to remove any outliers which can negatively influence the model fit. The filtered data included points which satisfy:

- CI ≥ 1
- CoV > 0
- Speed > 10km/h
- Free-flow speed (V_f) – 99th percentile of all speed values at a link within a month
- Weekdays, excluding public holidays
- No major incidents or extreme weather events.

The data was aggregated by each month for a given time interval for a link. This allowed the determination of day-to-day changes in link travel time within a 15-minute time interval on a month-by-month basis, which brings a fair bit of variability in the results as some months correspond to a particular traffic activity (e.g. December versus March). Combining several months data into one for analysis would have resulted in masking this phenomenon.

Similarly, the free-flow speed for a link was calculated for one month over a link. This implied that a total of four free-flow speeds were calculated for each link. For computation of free-flow speed, the 99th percentile of all speed values at a link within a month was used. This implied that only 1% of the data was above this threshold which mainly accounts for over-speeding vehicles and can safely be ignored. Free-flow travel time was then calculated as the ratio of link length and free-flow speed which corresponds to the 99th percentile speed.

Finally, CoV and CI are calculated which are fed into the model calibration procedure. A hypothetical example has been presented below which illustrates the steps followed to calibrate the ATAP link model.

- **Step 1 – The dataset:** The dataset (for a given road type: arterial or freeway) represents travel speed information for each link observed across 64 15-minute time periods (5am to 9pm) for all weekdays, excluding public holidays, in August to November 2018. For example, the figure below shows the prevailing speeds between 7:00-7:15am during weekdays, excluding public holidays, in August 2018 along with other information for link id 12 (that is the average speed of all vehicles recorded travelling on a link in a given 15-minute period and a day). Similarly, information for the other months for the same link can also be extracted.

LinkID	Time	Link Length (m)	Posted Speed (km/h)	Date	Prevailing Speed (km/h)
12	7-7:15	450	60	01-Aug-18	54
12	7-7:15	450	60	02-Aug-18	56
12	7-7:15	450	60	03-Aug-18	45
12	7-7:15	450	60	06-Aug-18	59
12	7-7:15	450	60	07-Aug-18	54
12	7-7:15	450	60	08-Aug-18	46
12	7-7:15	450	60	09-Aug-18	52
12	7-7:15	450	60	10-Aug-18	53
12	7-7:15	450	60	13-Aug-18	57
12	7-7:15	450	60	14-Aug-18	48
12	7-7:15	450	60	15-Aug-18	56
12	7-7:15	450	60	16-Aug-18	61
12	7-7:15	450	60	17-Aug-18	62
12	7-7:15	450	60	20-Aug-18	45
12	7-7:15	450	60	21-Aug-18	53
12	7-7:15	450	60	22-Aug-18	58
12	7-7:15	450	60	23-Aug-18	56
12	7-7:15	450	60	24-Aug-18	53
12	7-7:15	450	60	27-Aug-18	52
12	7-7:15	450	60	28-Aug-18	57
12	7-7:15	450	60	29-Aug-18	51
12	7-7:15	450	60	30-Aug-18	60
12	7-7:15	450	60	31-Aug-18	51

- **Step 2 – Free-flow speed and free-flow travel time:** The free-flow speed for each link during a month (weekdays, excluding public holidays) is calculated as the 99th percentile value from the prevailing speed data (across all time-periods). For example, the free-flow speed for link id 12 will be calculated as the 99th percentile of 1,472 prevailing speed observations (23 observations per time period multiplied by 64 15-minute time-periods) for the month of August 2018 (weekdays, excluding public holidays). Let us assume the calculated free-flow speed is 61km/h. Thus, the free-flow travel time, T_f, will be (450*60)/(1000*61) which is 0.442 minutes. Similarly, T_f for all other links and months can also be extracted.
- **Step 3 – Calculate travel time:** Calculate travel times by dividing the link length by the prevailing speeds. The figure below shows the conversion of data into travel time (in minutes) information. Similarly, travel time data can be obtained across all time-periods, links and months.

LinkID	Time	Link Length (m)	Posted Speed (km/h)	Date	Prevailing Speed (km/h)	Travel Time (mins)
12	7-7:15	450	60	01-Aug-18	54	0.5
12	7-7:15	450	60	02-Aug-18	56	0.48214
12	7-7:15	450	60	03-Aug-18	45	0.6
12	7-7:15	450	60	06-Aug-18	59	0.45763
12	7-7:15	450	60	07-Aug-18	54	0.5
12	7-7:15	450	60	08-Aug-18	46	0.58696
12	7-7:15	450	60	09-Aug-18	52	0.51923
12	7-7:15	450	60	10-Aug-18	53	0.50943
12	7-7:15	450	60	13-Aug-18	57	0.47368
12	7-7:15	450	60	14-Aug-18	48	0.5625
12	7-7:15	450	60	15-Aug-18	56	0.48214
12	7-7:15	450	60	16-Aug-18	61	0.44262
12	7-7:15	450	60	17-Aug-18	62	0.43548
12	7-7:15	450	60	20-Aug-18	45	0.6
12	7-7:15	450	60	21-Aug-18	53	0.50943
12	7-7:15	450	60	22-Aug-18	58	0.46552
12	7-7:15	450	60	23-Aug-18	56	0.48214
12	7-7:15	450	60	24-Aug-18	53	0.50943
12	7-7:15	450	60	27-Aug-18	52	0.51923
12	7-7:15	450	60	28-Aug-18	57	0.47368
12	7-7:15	450	60	29-Aug-18	51	0.52941
12	7-7:15	450	60	30-Aug-18	60	0.45
12	7-7:15	450	60	31-Aug-18	51	0.52941

- **Step 4 Mean and SD of travel time:** Calculate the mean and SD of travel time for each link, time-period and month (weekdays, excluding public holidays). Given that there are 64 15-minute time-periods to analyse (5am-9pm), there should be 64 mean and SD values per link in a month. For example, the mean and SD for link id 12 between 7-7:15am in August 2018 (weekdays, excluding public holidays) is 0.505 and 0.046 respectively. The SD has been calculated using the population standard deviation formula (e.g. STDEV.P function in MS-Excel).
- **Step 5 Compute CoV:** The CoV for each link, time period and month (weekdays, excluding public holidays) can be calculated as the ratio of SD of travel time and mean travel time. Thus, there will be 64 CoV values per link in a month. For example, the CoV for link id 12 between 7-7:15am in August 2018 (weekdays, excluding public holidays) is 0.046/0.505 which is 0.091.
- **Step 6 Compute CI:** The CI for each link, time period and month (weekdays, excluding public holidays) can be calculated as the ratio of prevailing travel time and free-flow travel time. Thus, there will be 64 CI values per link in a month. For example, the CI values for link id 12 between 7-7:15am in August 2018 (weekdays, excluding public holidays) are shown in the figure below. As mentioned in step 2, $T_f = 0.442$ minutes for link id 12 for August 2018.

LinkID	Time	Link Length (m)	Posted Speed (km/h)	Date	Prevailing Speed (km/h)	Travel Time (mins)	CI
12	7-7:15	450	60	01-Aug-18	54	0.5	1.13122
12	7-7:15	450	60	02-Aug-18	56	0.48214	1.09082
12	7-7:15	450	60	03-Aug-18	45	0.6	1.35747
12	7-7:15	450	60	06-Aug-18	59	0.45763	1.03536
12	7-7:15	450	60	07-Aug-18	54	0.5	1.13122
12	7-7:15	450	60	08-Aug-18	46	0.58696	1.32796
12	7-7:15	450	60	09-Aug-18	52	0.51923	1.17473
12	7-7:15	450	60	10-Aug-18	53	0.50943	1.15257
12	7-7:15	450	60	13-Aug-18	57	0.47368	1.07168
12	7-7:15	450	60	14-Aug-18	48	0.5625	1.27262
12	7-7:15	450	60	15-Aug-18	56	0.48214	1.09082
12	7-7:15	450	60	16-Aug-18	61	0.44262	1.00141
12	7-7:15	450	60	17-Aug-18	62	0.43548	0.98526
12	7-7:15	450	60	20-Aug-18	45	0.6	1.35747
12	7-7:15	450	60	21-Aug-18	53	0.50943	1.15257
12	7-7:15	450	60	22-Aug-18	58	0.46552	1.05321
12	7-7:15	450	60	23-Aug-18	56	0.48214	1.09082
12	7-7:15	450	60	24-Aug-18	53	0.50943	1.15257
12	7-7:15	450	60	27-Aug-18	52	0.51923	1.17473
12	7-7:15	450	60	28-Aug-18	57	0.47368	1.07168
12	7-7:15	450	60	29-Aug-18	51	0.52941	1.19776
12	7-7:15	450	60	30-Aug-18	60	0.45	1.0181
12	7-7:15	450	60	31-Aug-18	51	0.52941	1.19776

- **Step 7 Calculate (CI-1)/CI:** Calculate the independent variable in Equation 3.2
- **Step 8 Take natural logarithm:** Calculate the natural logarithm of CoV and (CI-1)/CI values computed in steps 5 and 7 respectively.
- **Step 9 Model fitting:** Estimate a linear regression model with Ln(CoV) as the dependent variable and Ln[(CI-1)/CI] as the independent variable. The result of model calibration will be Ln(a) and b values from Equation 3.2

Table 4-1 shows the calibration statistics for the two models, with respect to Equation 3.2, that is $\boldsymbol{Ln(CoV) = Ln(a) + b.Ln(\frac{CI-1}{CI})} \ \forall \ \boldsymbol{CI \geq 1}$, which was converted back into Equation 3.1, that is $\boldsymbol{CoV = a\left[\frac{(CI-1)}{CI}\right]^{b}} \ \forall \ \boldsymbol{CI \ \geq 1}$, the ATAP link Model.

As shown in the table, the p-value of the estimated parameters is less than 0.05 which implies that these are statistically significant at 95% confidence level across both models. The parameter Ln(a) corresponds to the intercept term in Equation 3.2 which was converted back into the parameter a in Equation 3.1 by taking an antilog. For example, if Ln(a) is equal to -0.521 for arterials, then a will be $e^{-0.521} = 0.5939$. The magnitudes of the calibration parameters (a and b) are lower for arterials when compared to freeways.

Considering a CI of 2, the CoV for arterials and freeways was found to be 0.30 and 0.37, respectively. Thus, the freeway dataset showed a higher travel time variability than the arterial dataset for a given CI level. This observation can be justified as follows: as the operating speeds of freeways are significantly higher, phenomena such as traffic oscillations occur at relatively lower congestion levels (than arterials) which lead to a spike in travel time variability.

The goodness-of-fit is measured by the Root Mean Squared Error (RMSE) value which is defined as the square root of the sum of squared differences between the observed and the predicted quantity, divided by the number of observations, that is, $RMSE = \sqrt{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2/n}$.

For the linear-log model given in Equation 3.2, the RMSE was found to be 0.4727 and 0.652 for arterial and freeway models respectively. Upon plugging the parameters (a and b) in Equation 3.1, the RMSE value with respect to Equation 3.1 was also calculated and found to be 0.1067 and 0.1235 for arterial and freeway models respectively.

The R-squared values for Equation 3.2 were found to be 0.559 and 0.668 for arterial and freeway models respectively. The R-squared values for Equation 3.1 were not calculated as these statistics do not truly convey the goodness-of-fit for non-linear models.

Table 4-1: ATAP link model calibration results

Parameter/Statistic	Arterial Model	Freeway Model
No. of observations in filtered dataset	162,301	79,655
Calibrated Parameters		
Ln(a)	-0.521*** (0.003) [-176.388]	-0.234*** (0.007) [-34.484]
a *(antilog of Ln(a))*	0.5939***	0.7913***
b	0.968*** (0.002) [453.793]	1.08*** (0.003) [400.759]
Goodness-of-Fit Statistics		
RMSE *(Ln(CoV))*	0.4727	0.652
RMSE *(CoV)*	0.1067	0.1235
R-squared *(Ln(CoV))*	0.559	0.668

*Note: asterisks denote statistical significance: * at 10%, ** at 5%, *** at 1%.*
Standard errors of parameters reported in (.).
T-statistics of parameters reported in [.].

In addition to the model represented by Equation 3.1 and 3.2, an alternative model specification was also tested. Equation 4.1 shows the alternative model form which represents a non-linear relationship between CoV and CI, with a and b as calibration parameters. By definition, the formula defines CoV as zero for $\mathrm{CI} \geq 1$. The CoV increases sharply at lower CIs, but eventually stabilises to a constant value for higher CIs. Equation 4.1 was calibrated on the same dataset using non-linear regression in the SPSS software package.

Table 4-2 shows the calibration results for Equation 4.1. Like Table 4-1, the parameters a and b are statistically significant at 95% confidence levels. The RMSE of the arterial and freeway models using Equation 4.2 is 0.1038 and 0.1154 respectively which indicates acceptable model fit to the observed data. An RMSE is an error around an observed CoV value. For example, an RMSE of 0.1 for the ATAP link model when it estimates a CoV value of 0.2, 0.3 or 0.4 suggests that the observed CoV would be on average around plus or minus 0.1 for the predicted value. This would be an acceptable level of precision for a link model.

Comparing these RMSE values (calculated with respect to CoV) with the ones reported in Table 4-1, the alternative model specification (Equation 4.1) is marginally better than the ATAP link model (Equation 3.1) for both arterials and freeways. In other words, both models yield similar levels of goodness-of-fit.

$$\boldsymbol{CoV = a.\left(1 - b^{(CI-1)}\right) \,\forall\, CI \geq 1} \quad \text{[EQ 4.1]}$$

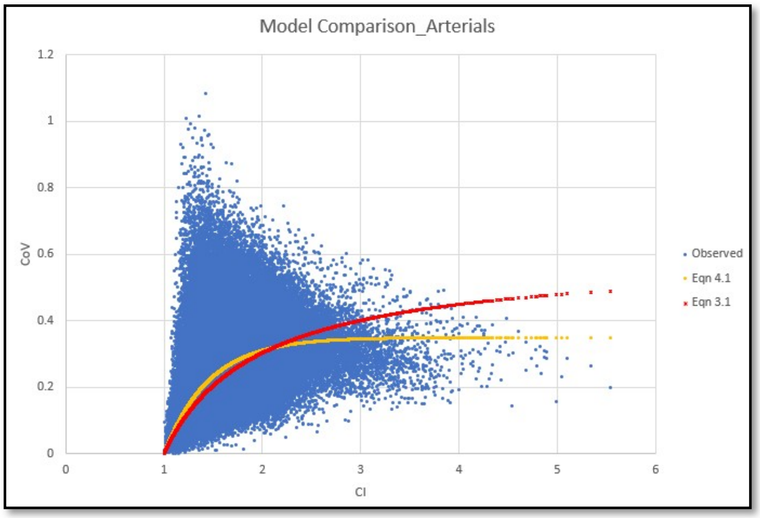
Table 4-2: Calibration results for the alternative model specification

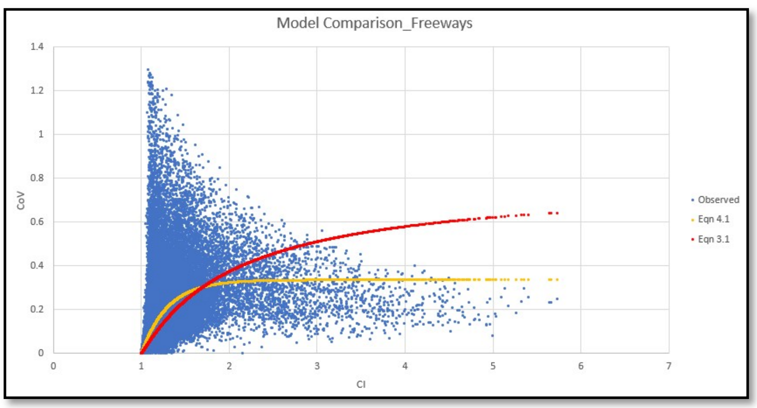
Parameter/Statistic	Arterial Model	Freeway Model
No. of observations (CI ≥ 1; CoV > 0; Speed > 10km/h)	162,301	79,655
a	0.35*** (0.001) [350.0]	0.336*** (0.002) [118.0]
b	0.112*** (0.001) [112.0]	0.036*** (0.001) [36.0]
RMSE *(CoV)*	0.1038	0.1154

*Note: asterisks denote statistical significance: * at 10%, ** at 5%, *** at 1%.*
Standard errors of parameters reported in (.).
T-statistics of parameters reported in [.].

Figure 4-1 shows the comparison of the models represented by the ATAP link model (Equation 3.1; in Red) and Equation 4.1 (in Yellow) on arterial and freeway datasets. Both models show a sharp rise in CoV for CI values up to 2. However, while Equation 4.1 stabilises and remains constant for higher CIs, the ATAP link model (Equation 3.1) continues to grow at a decaying rate. This means that Equation 4.1 will predict no improvement, that is, a constant travel time variability for any infrastructure changes or policies in areas with severe traffic congestion. This behaviour is considered counterintuitive as it is expected that minor improvements are possible in such scenarios. On the other hand, Equation 3.1, which is consistent with the law of diminishing returns, can account for gradual improvements in travel time variability at higher traffic congestion (CI values). Thus, Equation 3.1 was chosen as the preferred link model and addressed as the ATAP link model. Another reason for choosing Equation 3.1 over Equation 4.1 is Equation 3.1 can be fitted with ordinary least-squares regression of transformed data while Equation 4.1 requires non-linear regression. As well as being easier to implement, ordinary least squares regression provides an unambiguous, well understood R-squared measure of goodness of fit.

Figure 4-1: Link model comparison – NetPReS dataset





Thus, the calibrated ATAP models, based on the full NetPReS data are shown in Equations 4.2 and 4.3 [11].

$$\boldsymbol{Arterial}: \boldsymbol{CoV} = \mathbf{0.5939}.\left(\frac{\boldsymbol{CI}-\mathbf{1}}{\boldsymbol{CI}}\right)^{\mathbf{0.968}} \quad \text{[EQ 4.2]}$$

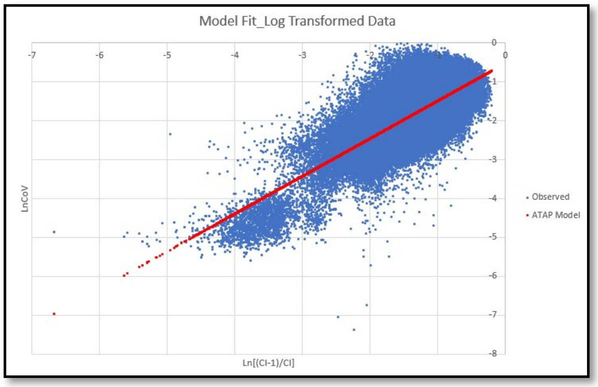
$$\boldsymbol{Freeway}: \boldsymbol{CoV} = \mathbf{0.7913}.\left(\frac{\boldsymbol{CI}-\mathbf{1}}{\boldsymbol{CI}}\right)^{\mathbf{1.08}} \quad \text{[EQ 4.3]}$$

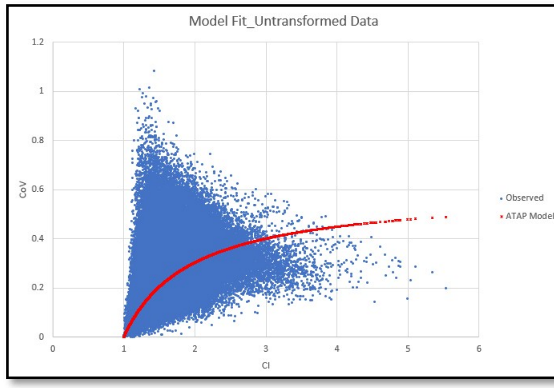
[11] For direct application of these models in calculations, compute the value of CI using the expression $\boldsymbol{CI} = \boldsymbol{max}(\mathbf{1}, \frac{\mathrm{T}}{\mathrm{T_f}})$ and use this value in Equations 4.1 and 4.2. This will automatically prevent the ATAP model from giving invalid CoV when CI < 1.

Figure 4-2 shows the ATAP link model fit on the NetPReS arterial and freeway data. For each road type, two plots are presented: (i) model fit on the log-transformed data (corresponding to Equation 3.2), and (ii) model fit on the actual data (that is, CoV versus CI). Both arterial and freeway plots show that the ATAP model link provides an acceptable goodness-of-fit, indicating its suitability in link travel time variability estimation.

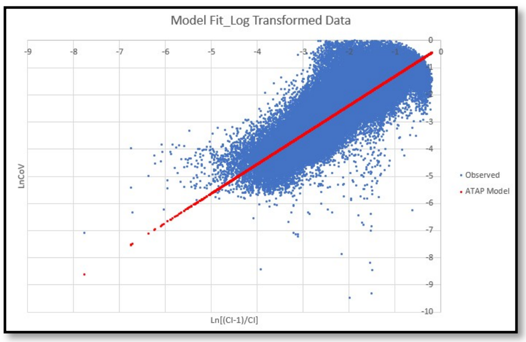
Figure 4-2: Travel time CoV vs CI with ATAP link model trendline – NetPReS dataset

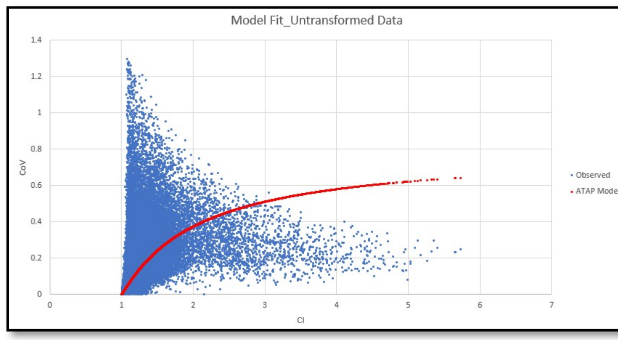
a) Arterial





b) Freeway





4.1.2 Validation of ATAP Link Model

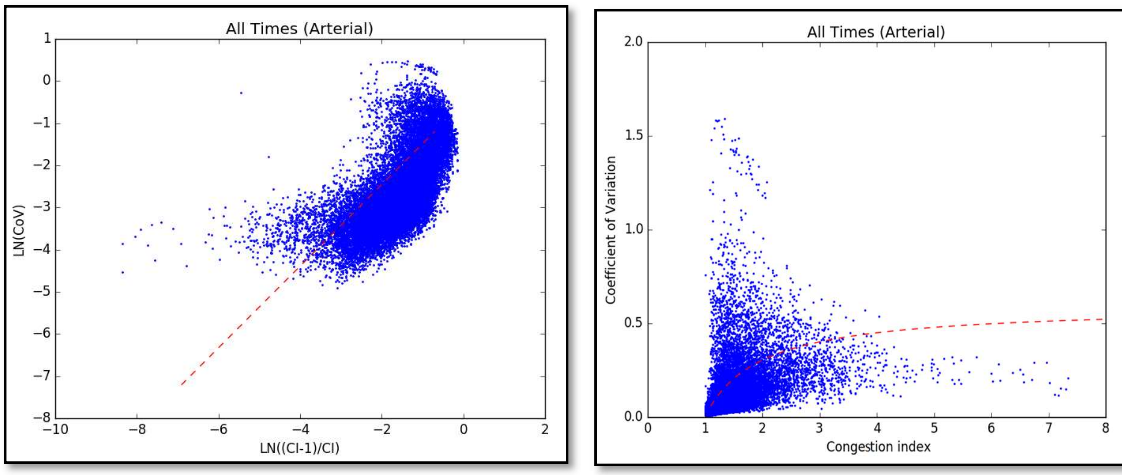
The ATAP link model was subsequently validated on Sydney Google data and Gold Coast Bluetooth data.

Sydney validation

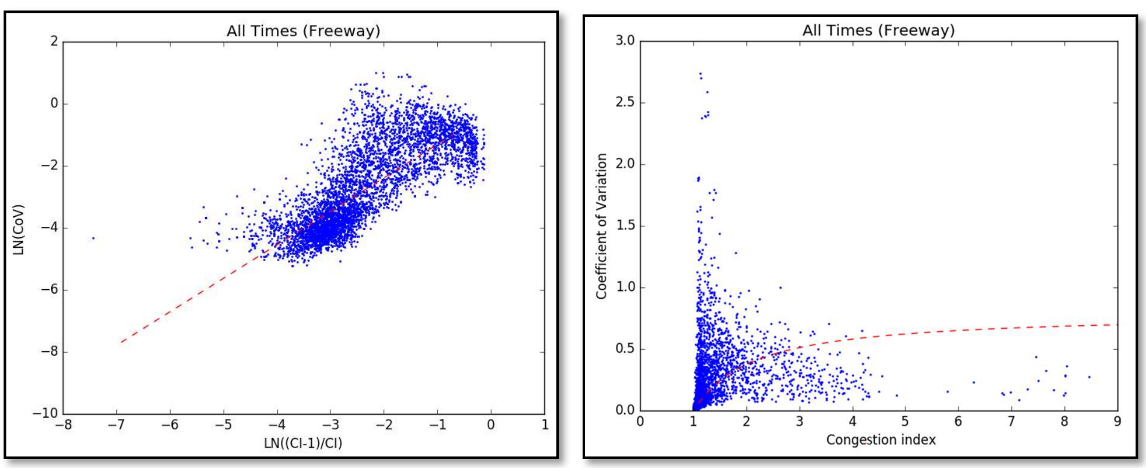
Figure 4-3 shows the ATAP link model fit on the Sydney Google travel time data. The figure shows model fit using the log-transformed and the actual data for both arterials and freeways. The plots show that ATAP link model provides an acceptable goodness-of-fit, indicating its suitability in link travel time variability estimation.

Figure 4-3: Travel time CoV vs CI with ATAP link model trendline – Sydney dataset

a) Arterial



b) Freeway



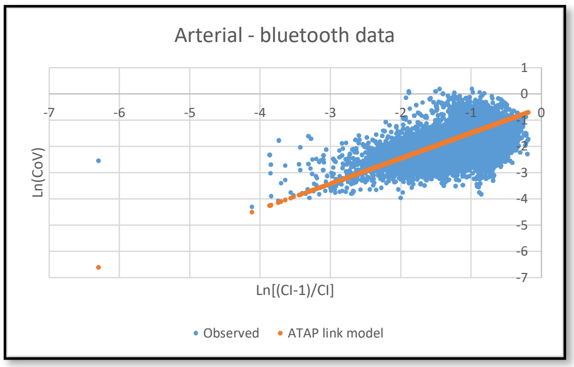
Gold Coast validation

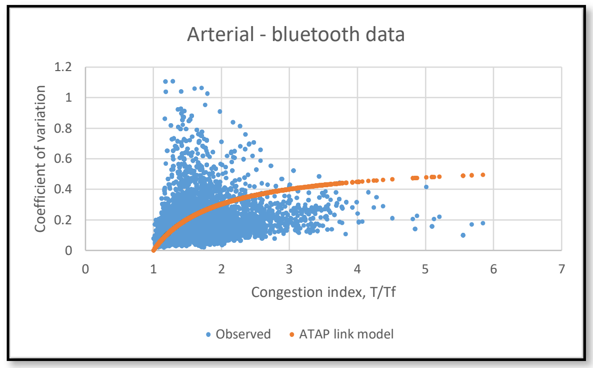
Figure 4-4 shows the ATAP link model fit on the Gold Coast Bluetooth travel time data. The arterial plots below show that the ATAP link model depicts an acceptable goodness-of-fit indicating its suitability in link travel time variability estimation. The freeway plots indicate a slightly over-estimated CoV for a given CI value.

This observation can be justified due to the geographical differences between the calibration Perth NetPReS dataset and the Gold Coast datasets resulting in differences in driving behaviour, road design and traffic conditions in those road networks. Nonetheless, the ATAP link model continues to forecast CoV at a reasonable level of accuracy making it the recommended modelling approach for estimating travel time variability on a link.

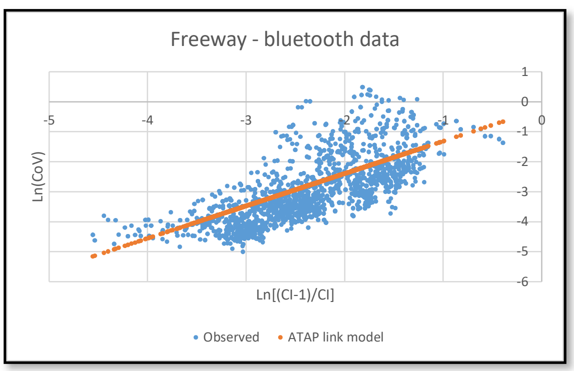
Figure 4-4: Travel time CoV vs CI with ATAP link model trendline – Gold Coast Bluetooth dataset

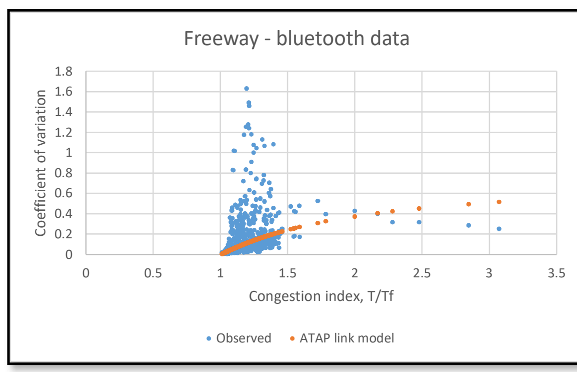
a) Arterial





b) Freeway





In summary, the results resented in the figures in this section demonstrate that the ATAP link model depicts an acceptable goodness-of-fit for the Perth, Sydney and Gold Coast datasets, indicating its suitability to be used as a national model in forecasting link travel time variability. The ATAP link model equations can be directly applied to traffic data from other jurisdictions. It is expected that the model would perform relatively well, with some level of variation such as the case in Gold Coast arterial network. Those variations are to be expected due to regional differences in network characteristics, travel behaviour and road conditions.

4.2 Route Travel Time Variability Model

According to Nicholson (2015) (as shown in Equation 2.3, that is $\boldsymbol{\sigma_r^2} = \sum_{i=1}^{n} \boldsymbol{\sigma_i^2} + \mathbf{2} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \boldsymbol{\rho_{i,j} \sigma_i \sigma_j}, i < j$), the variance of a route travel time is composed of two components:

1. The sum of the variances of all links travel time (the 'variance term') and
2. The sum of products of the travel time correlation and the SDs (the 'covariance term').

The route travel time variance was calculated by applying the ATAP link model to each individual link to obtain the CoV value and convert into travel time SD by using Equation 4.4:

$$\sigma_i = CoV_i \,.\, \mathrm{T}_i \qquad \text{[EQ 4.4]}$$

Where:

σ_i = SD of travel time on link i

CoV_i = travel time CoV on link i

T = mean travel time on link i (in minutes)

Therefore, the route travel time variability model calibration was focussed on the calibration of the CCM to account for the travel time correlation between any two links, including non-adjacent links.

4.2.1 Calibration of CCM

The linear-log CCM, shown in Equation 3.4 ($\boldsymbol{\rho_{i,j}} = \boldsymbol{Max}[\mathbf{0}, \boldsymbol{a} \,.\, \mathbf{Ln}(\mathbf{L}) + \mathbf{b}]$), was calibrated using the entire NetPReS dataset. The entire dataset was divided into segments based on the following classifiers: (i) road type (arterial and freeway), (ii) directionality (inbound and outbound), (iii) time of the day (AM, inter, PM and off-peak). This classification resulted in 16 data segments and the CCM was calibrated on each of them. The data used for analysis comprised travel time correlation between two links as the dependent variable and the mid-point distance (in kilometres) between the two links as an independent variable.

Apart from the CCM shown in Equation 3.4, two other model specifications, that is an exponential (Equation 3.3) and a shifted exponential (given by the equation $\boldsymbol{\rho_{i,j}} = \boldsymbol{a}\, \boldsymbol{e}^{\boldsymbol{b}\,\boldsymbol{L}} + \boldsymbol{c}$) were also tested, and the results compared in order to identify the best fitting model. The results showed that the linear-log CCM (Equation 3.3: $\boldsymbol{\rho_{i,j}} = \boldsymbol{a}\, \boldsymbol{Ln}(\boldsymbol{L}) + \boldsymbol{b}$) had the best R-squared value and statistically significant parameters across all 16 data segments. While the linear-log model was calibrated using linear regression, the other two functional forms were calibrated using non-linear regression (conducted in SPSS).

Table 4-3 shows the calibration statistics for the CCM:
(Equation 3.4, that is $\boldsymbol{\rho_{i,j}} = \boldsymbol{Max}[\mathbf{0}, \boldsymbol{a} \,.\, \mathbf{Ln}(\mathbf{L}) + \mathbf{b}]$).

As shown in the table, the p-value of the estimated parameters was less than 0.05 which implies that these are statistically significant at 95% confidence level across both models. The goodness-of-fit for freeway data was found to be higher when compared to that of arterials. Similarly, the parameters a and b were also found to be of greater magnitude for freeways when compared to arterials which indicate that freeways experience stronger travel time dependencies among links. This observation is sensible as freeway links are more directed and involve fewer physical impedances to movement that restrict links acting randomly from one another.

Once the correlation coefficient $\boldsymbol{\rho_{i,j}}$ between two links (within a route) are determined by the CCM (Equation 3.4), together with the ATAP link model (Equation 3.1) to estimate the travel time SD each link, the route travel time variance can be estimated by applying the CRM (Equation 2.3).

Table 4-3: Calibrated parameters for the proposed CCM

Road type	Direction	Time-period	a	b	R^2	RMSE
Arterial	Inbound	AM peak	-0.0482***	0.1658***	0.2148	0.1012
		Inter peak	-0.0236***	0.0638***	0.1248	0.0665
		PM peak	-0.0308***	0.0848***	0.1415	0.0961
		Off peak	-0.0445***	0.1590***	0.2239	0.1091
	Outbound	AM peak	-0.0302***	0.1076***	0.1176	0.0912
		Inter peak	-0.0234***	0.0631***	0.1460	0.0623
		PM peak	-0.0393***	0.1121***	0.2265	0.0838
		Off peak	-0.0391***	0.1362***	0.2083	0.0871
Freeway	Inbound	AM peak	-0.1098***	0.3477***	0.3476	0.1483
		Inter peak	-0.0870***	0.2653***	0.3129	0.1287
		PM peak	-0.0991***	0.3045***	0.3084	0.1473
		Off peak	-0.0992***	0.3128***	0.3285	0.1362
	Outbound	AM peak	-0.0620***	0.2078***	0.2286	0.1161
		Inter peak	-0.0745***	0.2293***	0.2871	0.1184
		PM peak	-0.1207***	0.4181***	0.3475	0.1710
		Off peak	-0.0979***	0.3539***	0.3248	0.1464

*Note: asterisks denote statistical significance: * at 10%, ** at 5%, *** at 1%.*

4.2.2 Validation of CRM – NetPReS

In order to validate the proposed method for estimating route reliability, measured route travel time standard deviations from the NetPReS data were compared with predictions from the correlation route model (CRM), that is $\boldsymbol{\sigma_r^2} = \sum_{i=1}^{n} \boldsymbol{\sigma_i^2} + \mathbf{2} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \boldsymbol{\rho_{i,j} \sigma_i \sigma_j}\ , i < j$. Figure 4-5 below illustrates the validation processes.

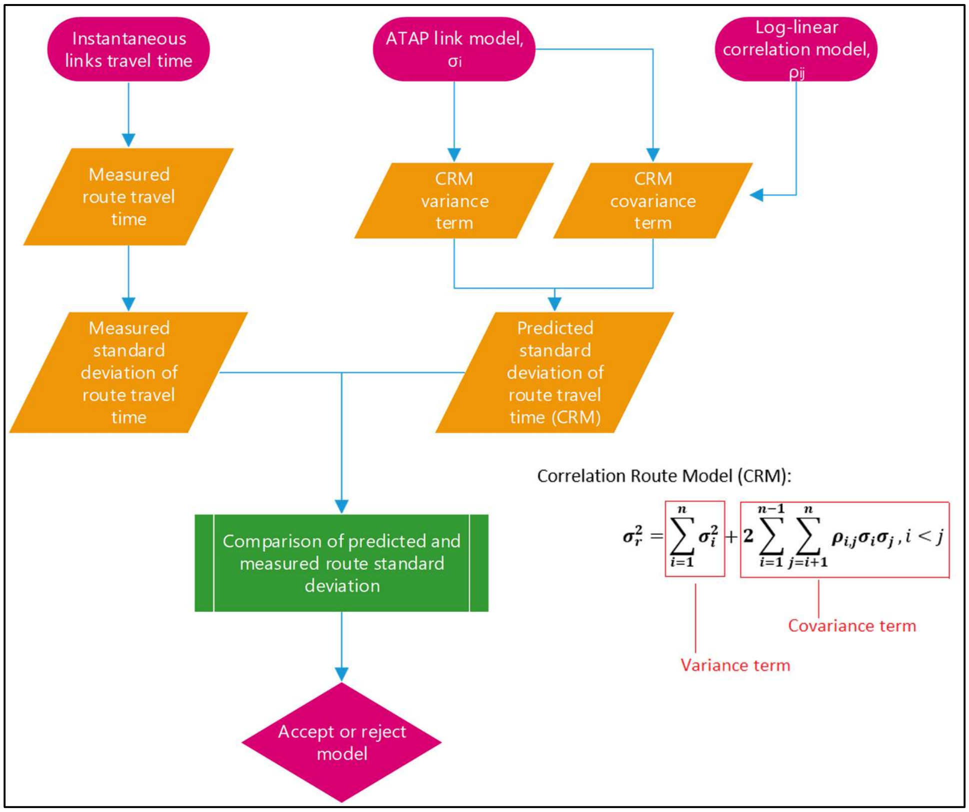
The sum of instantaneous link travel times constituting the route for a given day and 15-minute time period determined the route travel time. Similar route travel times were then calculated across weekdays, excluding public holidays, for the same time-period and the SD was then taken which represented the measured route travel time SD for that month. This is not the same as a single vehicle traversing the route because the time the vehicle entered each link during a trip would vary.

For example, the duration between 7am to 9am is comprised of eight 15-minute time periods. For each 15-minute time slice, the average link travel times were added together for each of the four months to obtain the route travel time. Eight route travel time observations were recorded for each month, and the process was repeated for the remaining time periods for each month by excluding weekends and public holidays within that month to create the entire route travel time dataset. The SD of route travel times could then be measured from the route travel time dataset for the given 15-minute time-period and the month.

The CRM, (Equation 2.3), can be described as the sum of a 'variance term', $\sum_{i=1}^{n} \boldsymbol{\sigma_i^2}$, and a 'covariance term', $\mathbf{2} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \boldsymbol{\rho_{i,j} \sigma_i \sigma_j}\ , i < j$.

The ATAP link model was used to obtain the link variances ($\boldsymbol{\sigma_i^2}$) that are summed in the variance term, as well as the SD ($\boldsymbol{\sigma_i}, \boldsymbol{\sigma_j}$) in the covariance term. The correlation parameter ($\boldsymbol{\rho_{i,j}}$) is determined using the CCM which provides a pair-wise correlation between two links within a route. The CCM (Equation 3.4) was fitted with simple linear regression.

Figure 4-5: Correlation route model validation flow chart



A total of 27 bi-directional routes available in the NetPReS dataset were utilised for validation of the CRM. The route set comprises 20 arterial routes and 7 freeway routes.

Figure 4-6 compares the measured and predicted route reliabilities for the arterial (left) and freeway (right) routes. The horizontal axis shows the measured route travel time SD and the vertical axis shows the route travel time SD predicted by the CRM. The four different coloured dot points indicate the four months that were used for analysis. Each dot point corresponds to a measured versus predicted comparison for a 15-minute time interval and a route.

While Figure 4-6 presents a comparison across all 64 15-minute time periods across four months, Figure 4-7 to Figure 4-10 presents the comparison (arterials on the left and freeways on the right) for the AM-peak (7am to 9am), Inter-peak (9am to 3pm), PM-peak (3pm to 6pm) and Off-peak (5am to 7am and 6pm to 9pm) periods respectively.

Figure 4-6: Route travel time SD validation across all time periods for arterials and freeways

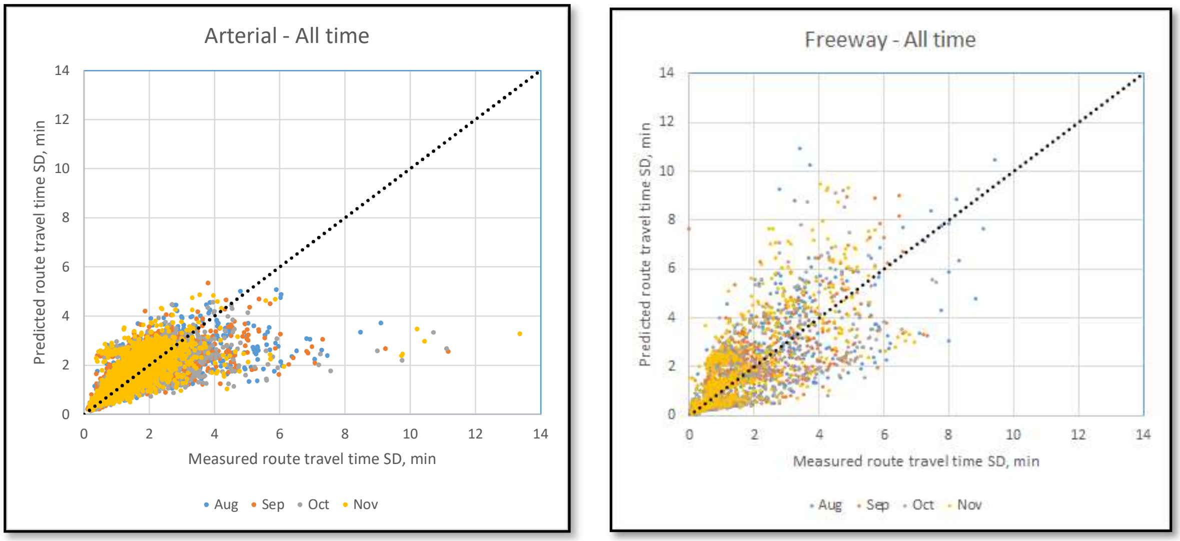


Figure 4-7: Route travel time SD validation across AM peak for arterials and freeways

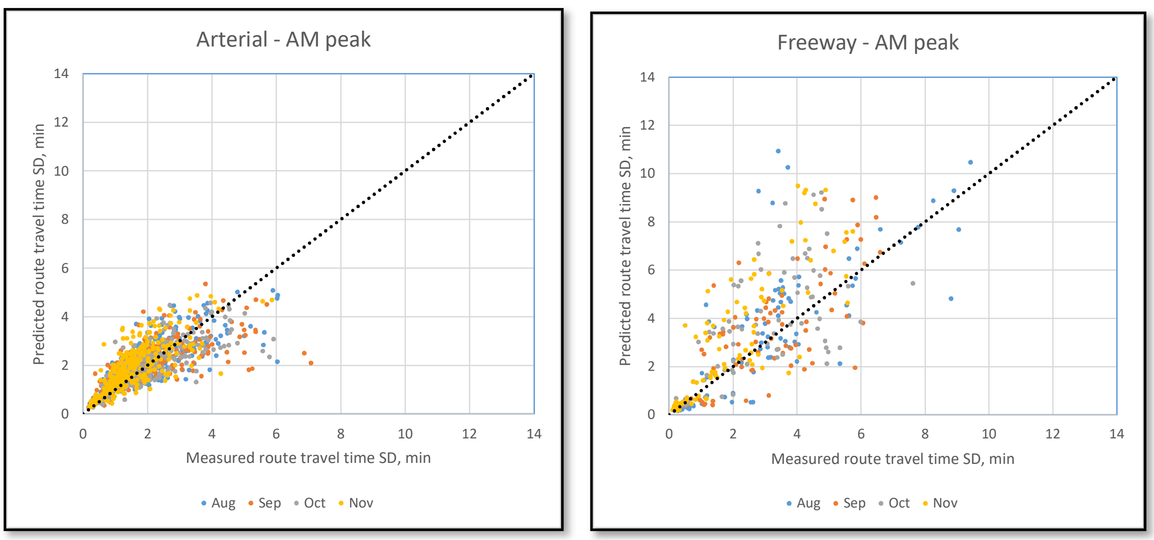


Figure 4-8: Route travel time SD validation across inter peak for arterials and freeways

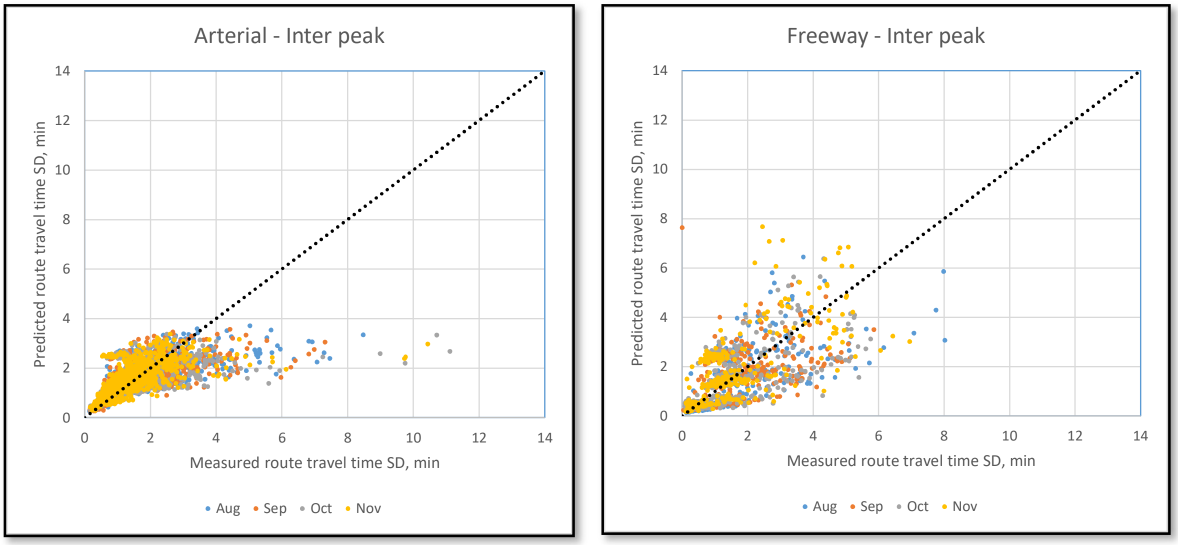


Figure 4-9: Route travel time SD validation across PM peak for arterials and freeways

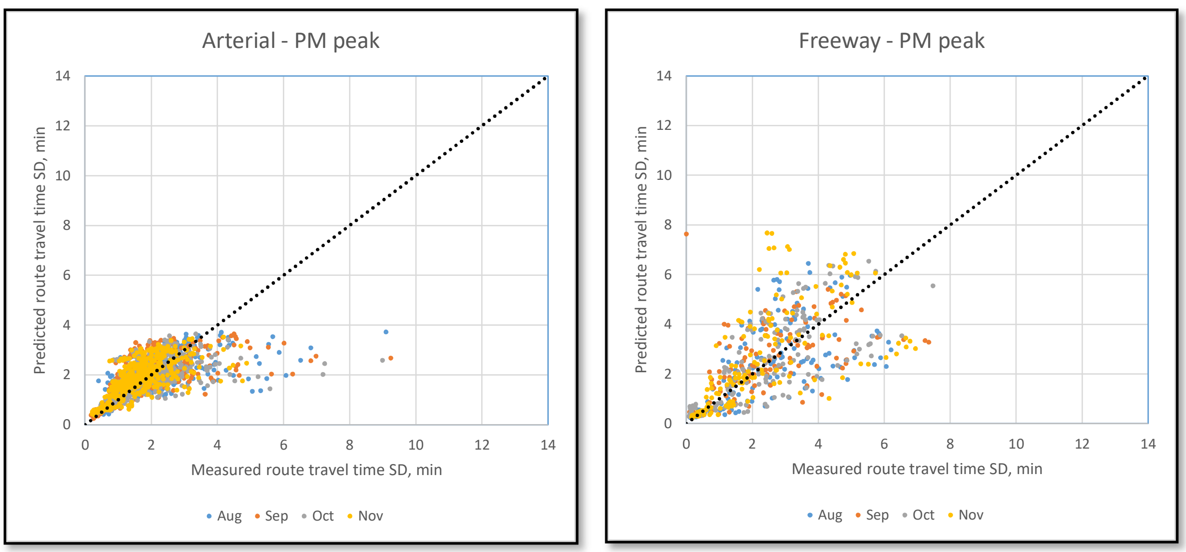
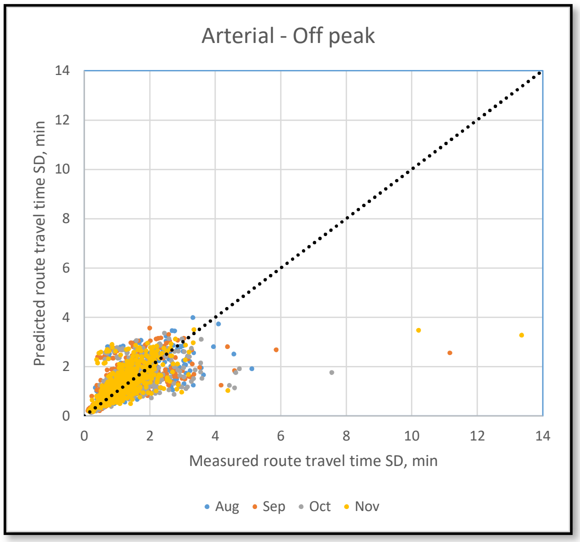
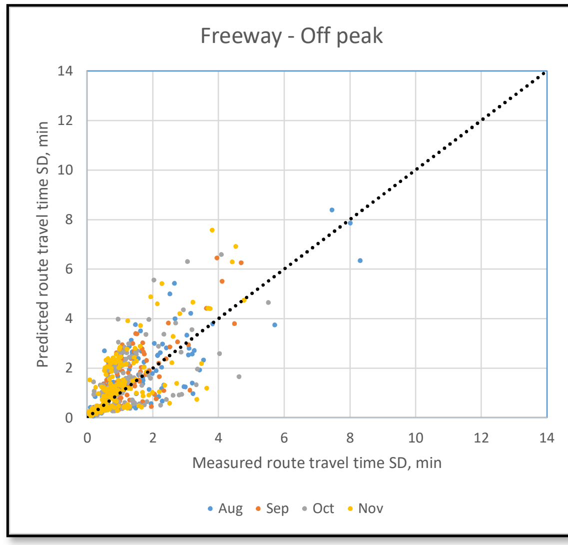


Figure 4-10: Route travel time SD validation across off peak for arterials and freeways





The 45-degree black dashed line shows a 1:1 relationship between the predicted and measured route travel time reliability values. A perfect model would have all dot points lying on the 1:1 dashed line. The greater the distance away from the dashed line, the greater the error in the travel time reliability prediction.

As Figure 4-7 to 4-10 show, the CRM gives a reasonable model fit to the measured route travel time SDs, with a majority of points clustered around the 1:1 (45-degree) trend line. The clustering around the dashed line is denser in the case of arterials when compared to freeways. Deviations from the 45-degree line can be explained by factors not taken into consideration by the CRM such as number of roundabouts and bottlenecks, geometric conditions, and negatively correlated links etc. which information was not available in the NetPReS dataset used for model development. Other factors such as incidents, weather, or events could also impact on the accuracy of the estimation. Given that the CRM comprises two sub models (ATAP Link Model and log-linear Correlation Model - Figure 4.5), it seems reasonable to suspect that most of the error in the CRM is due to inherent errors in these sub-models, and when models are applied together the error can compound.

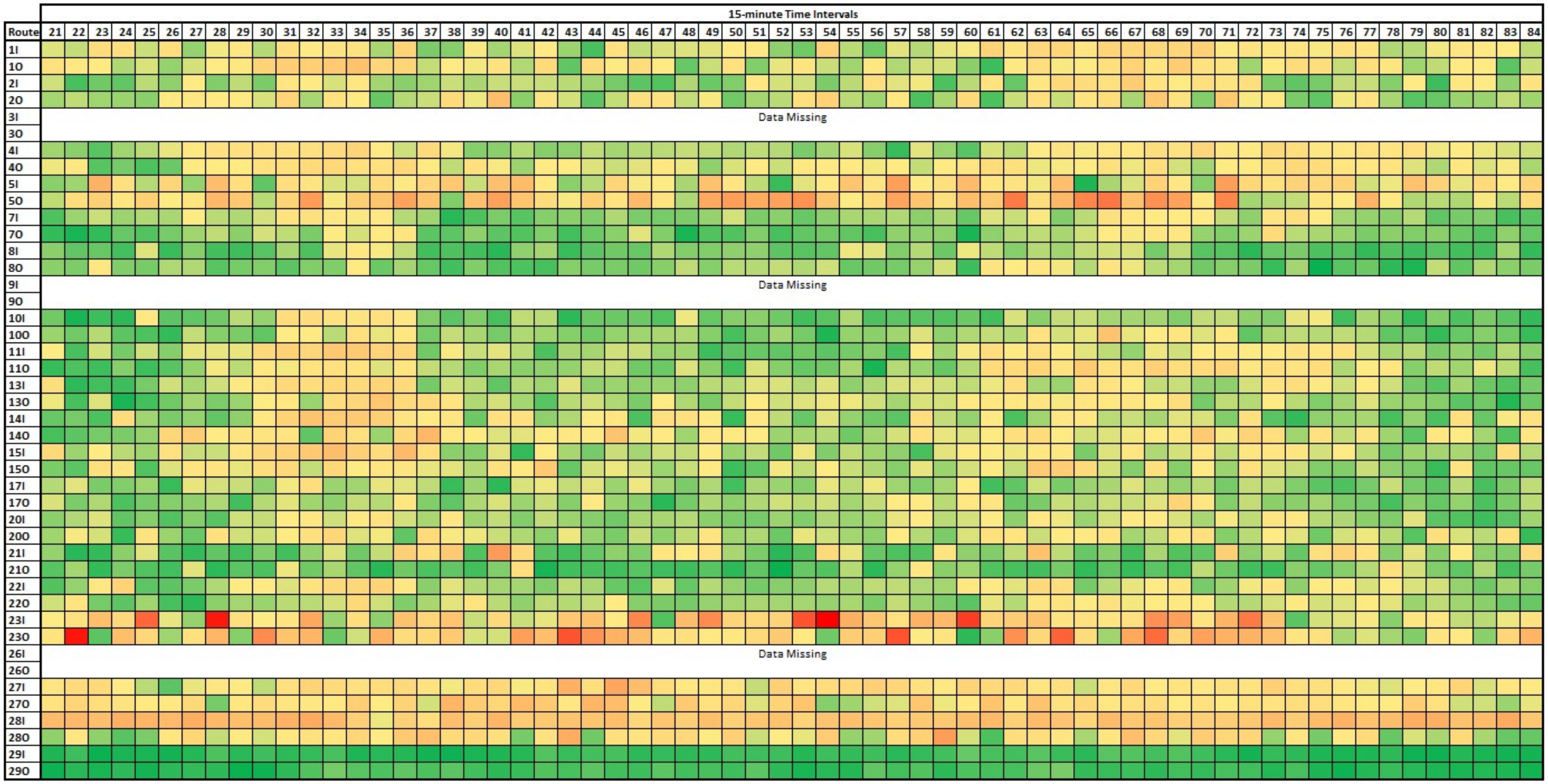
Figure 4-11 and Figure 4-12 present a heatmap of the RMSE values for arterials and freeways respectively. The rows in the heatmaps represent each route and direction with the number corresponding to the route id (corresponding route names provided in Appendix A3) and the suffix I and O representing whether the route is in the inbound and outbound direction respectively. The columns in the heatmaps correspond to 15-minute time periods within a day with the time-period id#21 corresponding to the time period 5:00 - 5:15am and id#84 corresponding to 8:45 - 9:00pm. The RMSE was calculated for each route and direction using the four measured and estimated route travel time SD across each of the 64 15-minute time periods.

The colour coding of the heatmaps is anchored around the following: the highest RMSE value (5.16) is denoted in Red, the lowest (0.01) in Green and an RMSE of 0.5 is coded in Yellow. In practice, an RMSE value of up to 0.5 signifies a decent model accuracy.

Based on this colour scheme, the CRM performs well (that is, not many Red cells which signify poor fit) for a majority of the routes (both arterial and freeway) across different time periods in a day. The blank rows in the heatmaps correspond to the routes which had missing data and were thus not populated in the heatmaps.

Figure 4-13 presents the frequency distribution of the RMSE values for arterials and freeways. The figure indicates median RMSE values of 0.40 minutes for arterials and 0.50 minutes for freeways, and average RMSE values of 0.52 minutes for arterials and 0.77 minutes for freeways. It implies that on average, the CRM produces an average dispersion of 0.52 or 0.77 minutes around the estimated route travel time variability. For example, an RMSE of 0.5 for a travel time variability value of 2 minutes, estimated by CRM, would suggest that the observed route variability value on average would be either 1.5 or 2.5 minutes which indicates good model precision.

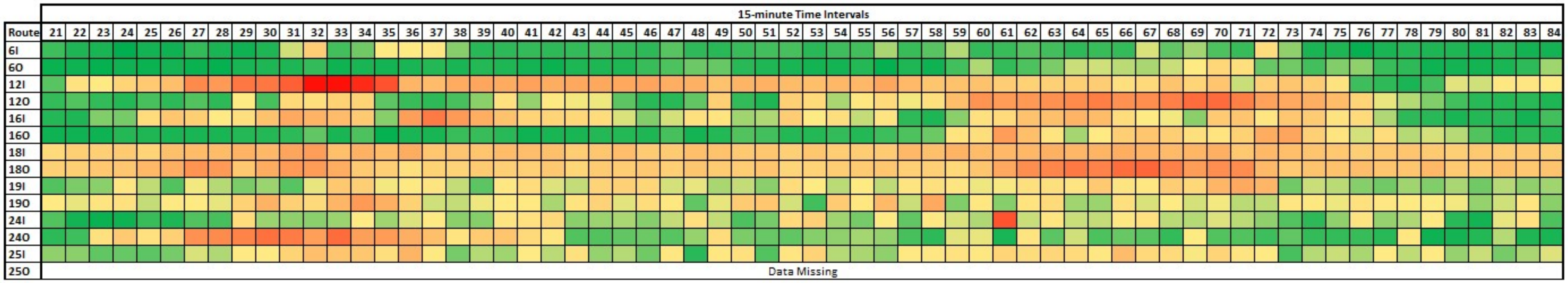
Figure 4-11: RMSE heatmap for arterials – NetPReS dataset



Legend: Green: Lowest RMSE; Red: Highest RMSE; Yellow: RMSE = 0.5.

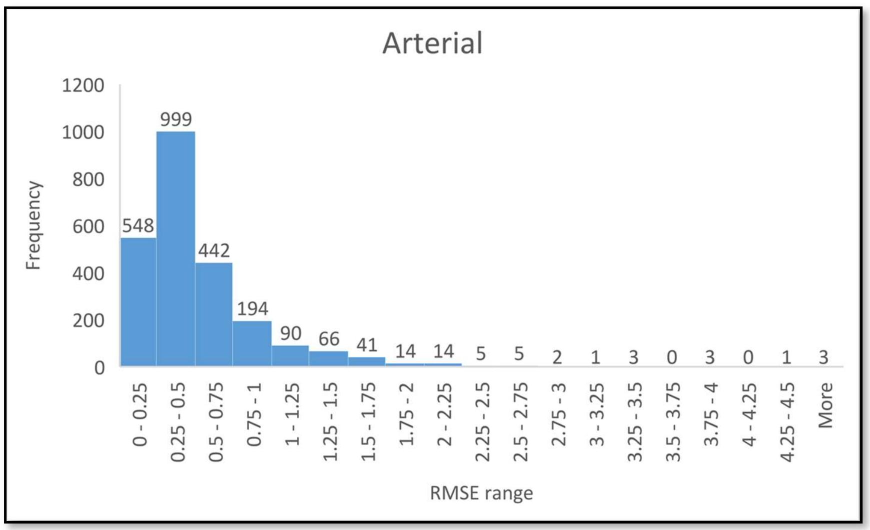
‘I’ refers to Inbound and ‘O’ refers to Outbound.

Figure 4-12: RMSE heatmap for freeways – NetPReS dataset



Legend: Green: Lowest RMSE; Red: Highest RMSE; Yellow: RMSE = 0.5.
'I' refers to Inbound and 'O' refers to Outbound.

Figure 4-13: RMSE frequency distribution



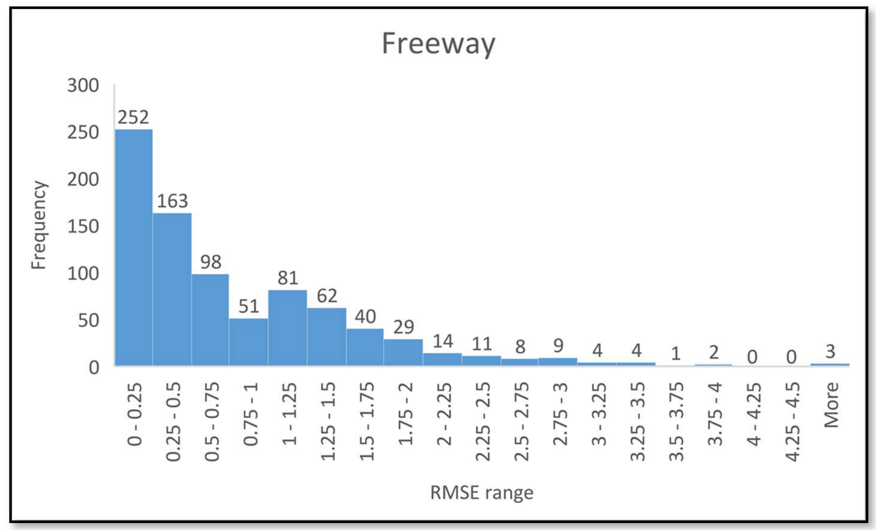


Figure 4-11 shows that most of the cells were Green and Yellow which indicates a lower RMSE for a majority of the routes across time periods, thus reporting that the CRM has a good model accuracy. This is consistent with the RMSE distribution in the arterial routes examined and shown in arterial distribution plot in Figure 4-13 where more than 50% of observations have RMSE values below 0.5 minutes.

The RMSE distribution in the freeway routes shown in heatmap of Figure 4-12 has a slightly higher proportion of Red cells which indicates a relatively lower model accuracy on freeway data. Similarly, Figure 4-13 shows that the median freeway RMSE value is slightly above 0.5 minutes. This could be due to limited freeway data availability which could influence the training (calibration) of the CRM.

The CRM has only been validated against the NetPReS data. Based on the link model validation presented in Section 4.1, it is expected that the current calibration parameters will have a reasonable goodness-of-fit on the Perth datasets. It is recommended that the parameters be recalibrated to fit other jurisdictional datasets.

In summary, the CRM is considered an appropriate methodology for forecasting route travel time variability as it not only considers independent link travel time variability (calculated using the ATAP link model in Equation 3.1), but also the variability which arises due to correlations among links forming a route (using the CCM in Equation 3.3).

The model was calibrated using the Perth NetPReS link dataset. The Perth route dataset, which was derived from the link dataset, was used to validate the CRM and showed a reasonable goodness-of-fit. Thus, this project recommends the use of the calibrated CRM (which includes the calibrated ATAP models (Equations 4.1 and 4.2) and CCM (Table 4.2)) to evaluate route travel time variability at a national scale. However, the CRM has been validated for NetPReS data only due to computational challenges associated in validation using other jurisdictional data. To use the CRM to measure travel time reliability changes in before-and-after cases for a specific route, practitioners and modellers should be confident that the CRM will capture the changes in travel time reliability with acceptable accuracy.

4.2.3 Application of CRM – Perth Arterial Case Study

The Perth case study focused on an evaluation of the Wanneroo Road Duplication project's impact on the travel time reliability during weekdays, excluding public holidays, by application of CRM. The Wanneroo Road Duplication project was a $31m project to widen Wanneroo Road, located at the northern side of Perth CBD running parallel to the Mitchell Freeway, from Joondalup Dr and Flynn Dr. This section was formerly a single carriageway carrying 26,000 vehicle per day. The project converted the single carriageway into dual carriageway in both directions between the section north of Joondalup Dr and the section south of Flynn Dr. The project commenced in November 2017 and was completed and open to traffic in April 2019.

Travel time reliability comparison was conducted based on the following criteria:

- Before period: August to October 2017 (Intelematics data)
- Alternative before period: August to October 2018 (NetPReS hybrid data)
- After period: August to October 2019 (NetPReS hybrid data)
- Time period: AM peak, 7am to 9am
- Temporal granularity: 15-minute
- Route: Wanneroo Road from Hester Ave to Ocean Reef Rd, a total length of 14,690 m
- Number of links: 8
- Direction: Inbound
- Exclusion: weekends, public holidays, major incident dates, and extreme weather dates.

Testing of the CRM on a project followed the steps outlined in Figure 4.5. It involves the steps of measuring and estimating day-to-day changes in the route travel time on weekdays, excluding public holidays, for before-and-after periods, and comparison of predicted route travel times against the measured route travel time. The box below outlines the steps undertaken in the testing of the CRM using data from this project.

Step 1:	Select one calendar month one 15-minute time-period data, 7.00 - 7.15am August 2018 for instance, apply CRM to estimate the route travel time SD for the selected time period.
Step 1.1:	Apply ATAP link model to estimate the travel time SD for each link. Application of ATAP link model requires the following inputs: free-flow travel time and mean travel time of the month for each link. Noted that free-flow travel time for a link will be constant for the selected month, however it may vary between different months.
Step 1.2:	Apply Linear-log correlation model to estimate the correlation coefficient between two links, includes non-adjacent links. Application of Linear-log correlation model requires the following inputs: mid-point to mid-point distance between two links, and selection of parameters for the time period of the day (e.g., AM peak) (from Table 4-3).
Step 1.3:	Apply CRM to estimate the route travel time SD. Application of CRM requires the following inputs: travel time SD for each links and correlation coefficient between two links, which are the outputs from Step 1.1 and Step 1.2 above.
Step 2:	Measure the 7am-7.15am route travel time SD from the field data.
Step 2.1:	For the given 7am-7.15am time period on each weekday (excluding public holidays) in August 2018, sum all links mean travel time to obtain the observed route travel time for the selected time period on that day.
Step 2.2:	There will be one route travel time value for each weekday (excluding public holidays) within the calendar month for the selected 15-minute time period. Take the SD value from those observed route travel time values, this would be the measure route travel time SD.
Step 3:	Repeat Step 1 and Step 2 above for each other 15-minute time periods, and for each calendar month within the before-and-after periods. This will give both measured SD and predicted SD for each 15-minute time period for each month, as shown in Table 4-4.
Step 4:	Calculate the changes in the measured route SD and predicted route SD for each 15-minute time period for each month, this will be the impact of the infrastructure project on the travel time SD. The comparison of values between measured SD and predicted SD give an indication of accuracy for CRM.
Step 5:	The average SD value or average change in SD in Table 4-4 is calculated by taking the mathematical average of relevant values above, e.g. $\bar{\sigma} = \sum_{1}^{24} \boldsymbol{\sigma_i} \div 24$. The difference between the average predicted route SD values for the before-and-after periods is the predicted travel time reliability impact of the project on the defined route inbound direction during AM peak period. The difference between the average measured route SD values for the before-and-after periods is the actual travel time reliability impact of the project on the defined route inbound direction during AM peak period. The comparison between the average measured change in route SD and the average predicted change in route SD gives the insight into the accuracy of application of CRM on assessing infrastructure project impacts on travel time reliability.

It was anticipated that the CRM would predict the route travel time SD with reasonable confidence in accuracy. The initial attempt to assess the accuracy of CRM by comparing the predicted route SD with the measured route SD showed that while the CRM predicted the after-period route SD with reasonable accuracy (Figure 4-14 (2), it underestimated the before period route SD significantly, as shown in Figure 4-14 (1) on the left hand side.

Investigation into the causes of underestimation in the before period travel times found that it was due to the before period data (Aug-Oct 2017) being a different data source (Intelematics) to the after period data (NetPReS hybrid data, which was sourced from multiple data providers such as TomTom, AddInsight, NPI, IRIS, and Intelematics). It was also noted that the CRM that was calibrated using NetPReS hybrid data and that should a different data source is used, then a recalibration of the model is required.

To address this inconsistency in the data sources for the before and after periods, the study examined an alternative before period (August to October 2018) when the NetPReS hybrid data was available and assessed the project's travel conditions at that time. Investigation of a series of high resolution historical aerial images of the construction sections of the study route from NearMap revealed that the travel condition was still single carriageway during the alternative before period. Therefore, it was possible for both the alternative before period and the after period to be assessed using the single NetPReS dataset. Five mean speed observations below 20km/h, which were considered as construction impact, were removed from raw datasets.

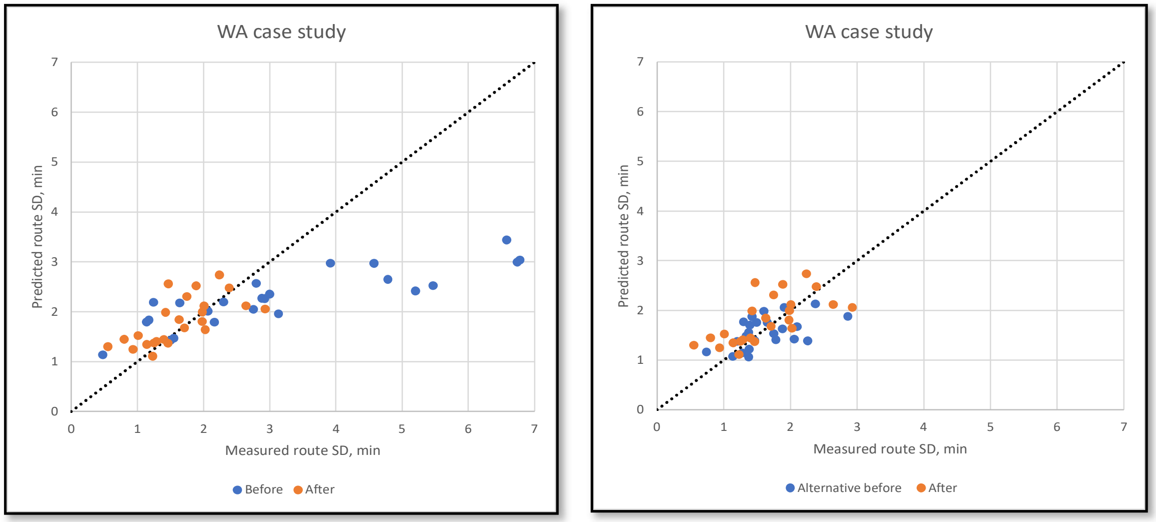
Figure 4-14 shows the visual comparison of the measured and predicted route SDs for both before-and-after periods (using Intelematics for before period and NetPReS for the after period) and alternative-before-and-after periods (using only the single NetPReS dataset). While the CRM underestimated the before period route SD due to the speed data came from the different source, it produced a reasonable amount of accuracy in the predicted route SD values for both the before period and after period when only the NetPReS data was used.

Table 4-4 shows the measured versus predicted route SDs for the alternative before period and after period and its changes. Two methods of calculating route SD are presented: the predicted route SD using CRM and the measured route SD from field data. The comparison of the predicted changes in route SD with the measured changes in route SD in the before-and-after periods analysis give an indication of route SD prediction accuracy from CRM. Therefore, the average measured and predicted route SD values for before-and-after periods are compared.

In summary, CRM predicts that the Wanneroo Road Duplication project would increase the route travel time SD by 0.3 minutes on average for AM peak inbound direction of the route. This compares to the measured average change of 0.0 minutes in route travel time SD from the field data. CRM overestimates the change in travel time reliability by 0.3 minutes per vehicle.

For the alternative before period, the average route SD value CRM predicted matches the average route SD value measured from field data. For the after period, the CRM on average overestimates the route SD by 0.2 minutes or 12.5%.

Figure 4-14: Measured versus predicted route SD – Perth arterial case study



(1) Before period – Intelematics data

(2) Alternative before period – MRWA hybrid data

Table 4-4: Measured versus predicted route SD – Perth arterial case study

Time	Route SD alternative before period, min						Route SD after period, min						Changes in route SD, min					
	Aug		Sep		Oct		Aug		Sep		Oct		Aug		Sep		Oct	
	SDm[1]	SDp[2]	SDm	SDp	SDm	SDp	SDm	SDp	SDm	SDp	SDm	SDp	ΔSDm	ΔSDp	ΔSDm	ΔSDp	ΔSDm	ΔSDp
07:00 - 07:15	2.0	1.4	1.1	1.1	1.3	1.1	1.2	1.1	1.2	1.4	0.9	1.2	-0.8	-0.3	0.1	0.3	-0.4	0.2
07:15 - 07:30	2.8	1.9	1.2	1.1	0.7	1.2	1.4	1.4	1.7	1.7	1.3	1.4	-1.4	-0.4	0.4	0.5	0.5	0.2
07:30 - 07:45	1.4	1.7	1.3	1.2	1.3	1.5	1.9	2.0	1.4	2.0	1.9	1.8	0.6	0.3	0.1	0.8	0.6	0.3
07:45 - 08:00	1.8	2.1	1.2	1.4	2.2	1.4	1.7	2.3	1.4	2.6	2.6	2.1	-0.1	0.3	0.3	1.2	0.4	0.7
08:00 - 08:15	1.6	2.0	1.7	1.4	1.4	1.4	2.2	2.7	1.8	2.5	2.9	2.1	0.6	0.8	0.1	1.1	1.4	0.7
08:15 - 08:30	1.4	1.8	1.7	1.5	2.0	1.7	2.3	2.5	2.0	2.1	2.0	1.6	0.9	0.7	0.3	0.6	-0.1	0.0
08:30 - 08:45	1.4	1.9	1.6	1.8	1.3	1.8	1.6	1.8	0.8	1.4	1.4	1.4	0.2	0.0	-0.8	-0.3	0.2	-0.4
08:45 - 09:00	2.3	2.1	1.3	1.6	1.8	1.6	1.0	1.5	0.5	1.3	1.1	1.3	-1.3	-0.6	-0.8	-0.3	-0.7	-0.3
Average measured	1.6						1.6						0.0					
Average predicted	1.6						1.8						0.3					

[1] SDm refers to the measured route SD in minute.

[2] SDp refers to the predicted route SD in minute.

4.2.4 Application of CRM – Brisbane Freeway Case Study

The Brisbane case study focuses on evaluation a series of infrastructures projects on Bruce Hwy between 2015 to 2019 and their impacts on Bruce Hwy inbound travel time reliability by application of CRM. Those projects include:

- Managed motorway treatments, it includes ramp signalling at five locations in the study route, variable speed limit (VSL) signs and queue detection/queue protection systems. Those systems were activated at different time from September 2015 onwards and fully activated by the end of 2016.
- Boundary Rd interchange, it involved the upgrade of the Boundary Road interchange approximately 30 km north of the Brisbane CBD and included a new six lane, four span concrete bridge over the Bruce Highway. Construction began in May 2016 and opened to service on 8 September 2017.
- Gateway Upgrade North, it involved the upgrade of the Gateway Motorway between Nudgee and Deagon, with additional pavement and safety works through to Bracken Ridge. Major construction started in February 2016 and completed in March 2019. The project is expected to ease the congestion experienced at downstream of Bruce Hwy inbound before the Gateway Motorway.

It is worth noting that the actual morning peak on Bruce Hwy spans between 5 am and 10 am, however for the purpose of CRM testing, the analysis period was kept at typical AM peak between 7 am and 9 am during weekdays, excluding public holidays. Travel time reliability comparison was conducted based on the following criteria:

- Before period: June to August 2015
- After period: June to August 2019
- Time period: AM peak, 7 am to 9 am
- Temporal granularity: 15 min
- Route: Bruce Hwy between Bribie Island Rd and Bracken Ridge, a total length of 28,346 m
- Number of links: 17
- Direction: Inbound
- Exclusion: weekends, public holidays, major incident dates, and extreme weather dates.

By following the same steps in Section 4.2.3, Figure 4-15 shows the visual comparison of the measured and estimated route SDs. Unlike the Perth case study which showed reasonably accurate estimates compared to measured values, the Brisbane route SD estimate are significantly lower than the measured values. Table 4-5 shows the measured versus predicted route SDs and the changes in route SDs in before-and-after periods.

Figure 4-15: Measured versus predicted route SD – Brisbane freeway case study

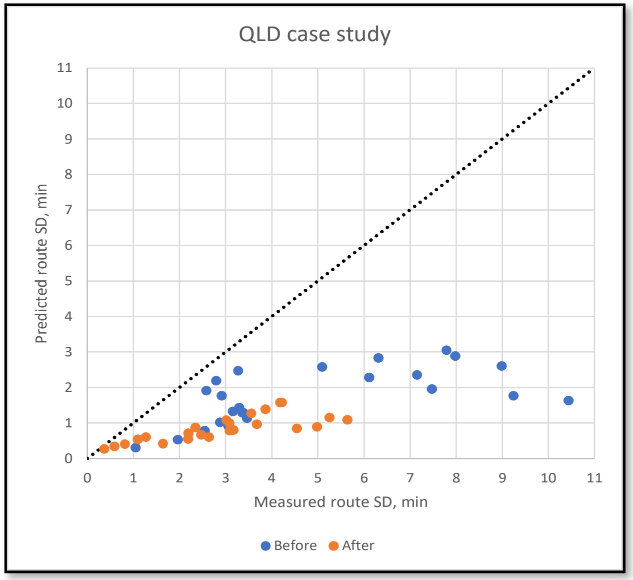


Table 4-5: Measured versus predicted route SD – Brisbane freeway case study

Time	Route SD alternative before period, min						Route SD after period, min						Changes in route SD, min					
	Aug		Sep		Oct		Aug		Sep		Oct		Aug		Sep		Oct	
	SDm	SDp	SDm	SDp	SDm	SDp	SDm	SDp	SDm	SDp	SDm	SDp	∆SDm	∆SDp	∆SDm	∆SDp	∆SDm	∆SDp
07:00 - 07:15	6.1	2.3	3.5	1.1	9.0	2.6	3.1	0.8	0.6	0.3	3.1	1.0	-3.0	-1.5	-2.9	-0.8	-5.9	-1.6
07:15 - 07:30	6.3	2.8	3.4	1.3	5.1	2.6	3.2	0.8	2.2	0.5	3.6	1.3	-3.1	-2.0	-1.2	-0.7	-1.5	-1.3
07:30 - 07:45	7.8	3.0	3.2	1.3	3.3	2.5	3.7	1.0	4.5	0.8	4.2	1.6	-4.1	-2.1	1.4	-0.5	0.9	-0.9
07:45 - 08:00	8.0	2.9	3.4	1.3	2.8	2.2	5.3	1.2	2.5	0.7	4.2	1.6	-2.7	-1.7	-0.9	-0.6	1.4	-0.6
08:00 - 08:15	7.1	2.4	2.9	1.0	2.6	1.9	5.6	1.1	1.3	0.6	3.9	1.4	-1.5	-1.3	-1.6	-0.4	1.3	-0.5
08:15 - 08:30	7.5	2.0	2.5	0.8	2.9	1.8	5.0	0.9	1.1	0.5	3.0	1.1	-2.5	-1.1	-1.5	-0.2	0.1	-0.7
08:30 - 08:45	9.2	1.8	2.0	0.5	3.3	1.4	2.6	0.6	0.8	0.4	2.3	0.9	-6.6	-1.2	-1.1	-0.1	-1.0	-0.6
08:45 - 09:00	10.4	1.6	1.1	0.3	3.1	0.9	1.6	0.4	0.4	0.3	2.2	0.7	-8.8	-1.2	-0.7	0.0	-0.9	-0.2
Average measured	4.8						2.9						-1.9					
Average predicted	1.8						0.8						-0.9					

[1] SDm refers to the measured route SD in minute.

[2] SDp refers to the predicted route SD in minute.

In summary, CRM predicts that those past infrastructure projects on Bruce Hwy Queensland between 2015 and 2019 would reduce the route travel time SD by 0.9 min in average for AM peak inbound direction of the study route. This compares to the measured reduction of 1.9 min in route travel time SD from the field data, CRM underestimates the actual benefit in reliability cost saving by 1 min per vehicle, or 52.6%. On average, the CRM underestimates the before period and after period route SDs by 62.5% and 72.4% respectively.

There were three main reasons that caused this underestimation:

1. The ATAP link model and CCM parameters were calibrated based on Perth NetPReS hybrid data, while the Brisbane case study was based on a single source of data. Different data collection method behaves different with their own way of generating errors. Therefore, local calibration is recommended, and practitioners may also consider what is the predominant methods of data being collected.

2. The underestimation may be caused by the geographical differences between the calibrated Perth NetPReS dataset and the Brisbane datasets. This geographical difference may result in differences in driving behaviour, road design and traffic conditions on those road networks. Recalibration of the ATAP link model using Queensland data may solved this underestimation.

3. It can also be the nature of the predicted model that there will always be some level of overestimation or underestimation which is consider as error in the model. Using Perth network as an example in Figure 4-6, by applying the CRM, some routes may show very accurate estimates while others may show significant over-or-underestimations like the Bruce Hwy estimates. Therefore, unless the model is calibrated on route-by-route basis, this type of error cannot be eliminated. The model can however be improved to reduce this error by further study into other variables which also have impacts on travel time reliability and were not considered in this model.

4.3 Network Travel Time Variability Models

4.3.1 Calibration of ARSD

The calibration of ARSD model involves determining the value of $\boldsymbol{\gamma}$ in Equation 3.5 (that is

$\boldsymbol{\sigma_r} \approx \boldsymbol{\gamma} \sum \boldsymbol{\sigma_l}$) based on the real-world information.

Investigation into the relationship between route travel time SD and $\boldsymbol{\gamma}$ has concluded that the level of travel time correlation between links determines the value of $\boldsymbol{\gamma}$, and it varies with the time-period, direction of travel, length and road type. Due to the complexity of its relationship between the $\boldsymbol{\gamma}$ value and its dependent variables, and its proposed use as an approximation only, the calibration of ARSD model was not considered within the scope of purposes and it is not conducted in this project.

The entire NetPReS dataset was used to calculate the likely $\boldsymbol{\gamma}$ value range. The NetPReS dataset was segregated into two segments: arterials and freeways. The value $\boldsymbol{\gamma}$ for each route was calculated as the ratio of the route travel time SD (obtained from the CRM in Equation 2.3) and link travel time SD (obtained from the ATAP link model in Equation 3.1). The travel time SDs were determined for the AM peak period only. Table 4-6 shows the route-specific $\boldsymbol{\gamma}$ values along with other characteristics. The fifth column in the table gives the resulting $\boldsymbol{\gamma}$ value for each considered route. The last column shows the covariance term of the CRM as a percentage of route variance. It gives the proportion of route travel time variance accounted for by correlation between links, ranging from 6% to 71%. This shows the importance of travel time correlation between links and that it certainly cannot be ignored.

Table 4-6: Estimated γ value from available routes in Perth network and its level of correlation between Links

Route	Direction	Length, km	Road type	Gamma AM	AM covariance as % of route
Albany Hwy	Inbound	51.2	Arterial	0.34	63%
Albany Hwy	Outbound	51.2	Arterial	0.30	53%

Route	Direction	Length, km	Road type	Gamma AM	AM covariance as % of route
Armadale Rd	Inbound	15.88	Arterial	0.43	43%
Armadale Rd	Outbound	15.88	Arterial	0.40	33%
Canning Hwy	Inbound	16.26	Arterial	0.39	58%
Canning Hwy	Outbound	16.26	Arterial	0.34	49%
Graham Farmer Fwy	Inbound	6.81	Freeway	0.67	44%
Graham Farmer Fwy	Outbound	6.81	Freeway	0.53	48%
Great Eastern Hwy Inner	Inbound	13.94	Arterial	0.40	61%
Great Eastern Hwy Inner	Outbound	13.94	Arterial	0.35	53%
Great Eastern Hwy Outer	Inbound	47.53	Arterial	0.47	48%
Great Eastern Hwy Outer	Outbound	47.53	Arterial	0.42	40%
Guildford Rd	Inbound	11.01	Arterial	0.43	53%
Guildford Rd	Outbound	11.01	Arterial	0.39	42%
Karrinyup-Morley Hwy	Inbound	15.12	Arterial	0.40	59%
Karrinyup-Morley Hwy	Outbound	15.12	Arterial	0.36	50%
Kwinana Fwy	Inbound	57.03	Freeway	0.48	66%
Kwinana Fwy	Outbound	57.03	Freeway	0.30	52%
Leach Hwy	Inbound	23.6	Arterial	0.36	57%
Leach Hwy	Outbound	23.6	Arterial	0.34	46%
Marmion Av	Inbound	12.2	Arterial	0.51	30%
Marmion Av	Outbound	12.2	Arterial	0.47	24%
Melville Mandurah Hwy	Inbound	48.23	Arterial	0.30	51%
Melville Mandurah Hwy	Outbound	48.23	Arterial	0.27	45%
Mitchell Fwy	Inbound	35.04	Freeway	0.43	71%
Mitchell Fwy	Outbound	35.04	Freeway	0.33	64%
Orrong Rd	Inbound	10.16	Arterial	0.45	52%
Orrong Rd	Outbound	10.16	Arterial	0.42	36%
Reid Hwy	Inbound	21.39	CAH	0.48	60%
Reid Hwy	Outbound	21.39	CAH	0.42	50%
Roe Hwy	Inbound	34.09	CAH	0.41	59%
Roe Hwy	Outbound	34.09	CAH	0.38	48%
South St	Inbound	12.48	Arterial	0.43	53%
South St	Outbound	12.48	Arterial	0.40	39%
Stirling Hwy	Inbound	13.85	Arterial	0.45	51%
Stirling Hwy	Outbound	13.85	Arterial	0.37	47%
Thomas Rd	Inbound	18.5	Arterial	0.70	7%
Thomas Rd	Outbound	18.5	Arterial	0.68	6%
Tonkin Hwy North	Inbound	7.68	CAH	0.79	13%
Tonkin Hwy North	Outbound	7.68	CAH	0.75	15%
Wanneroo Rd / Indian	Inbound	59.17	Arterial	0.34	58%
Wanneroo Rd / Indian	Outbound	59.17	Arterial	0.30	50%
West Coast Hwy	Inbound	14.09	Arterial	0.55	24%
West Coast Hwy	Outbound	14.09	Arterial	0.41	33%
Arterial median				0.41	49%
Freeway median				0.45	51%

Since the ARSD model is an approximation intended for use in network assignment (route choice) when accuracy is not critical, the γ values in Table 4-6 give modellers an indication of the gamma value range observed for the Perth network. The table indicates a median γ value for arterials and freeways as 0.41 and 0.45 respectively. These values can be applied to other networks for network assignment, but only in cases when an approximation is fit for purpose. The numerical analysis in Appendix A2.3 concluded the potential impact of a constant γ value across network: "*The correction factor (γ) is somewhat sensitive to the correlation between the links… If the correlation between the links is more or less consistent across the network, then the impact of the level of correlation on accuracy would be marginal.*".

In the case where a constant γ value is not suitable for its use, one of the following alternative approaches could be adopted:

- Observed the γ value from field data on a route-by-route basis.
- Use the CRM to estimate the gamma value on a route-by-route basis.
- Use StrUE as an alternative network travel time variability model.

4.3.2 Application of ARSD – Sydney Case Study

The ARSD model was applied to the link specific traffic information for the AM peak period in the Sydney dataset, which comprises 74 (37 routes times bidirectional flow) arterial and freeway routes (refer to Appendix A3 for details of the Sydney data). The link level travel time SD is initially computed using the ATAP link model for arterial (Equation 4.1) and freeway (Equation 4.2). It is then multiplied by the median γ values of 0.41 and 0.45 to obtain the route travel time SD.

Figure 4-16 and Figure 4-17 show the inbound and outbound travel time SD for the arterial and freeway routes respectively. The figures show the travel time variability for the arterials and freeway routes considered in this case study. The plots show that the routes with the highest travel time SD in both the directions are route 8 (A34) for arterials and route 3 (M5) for freeways (see Table A-4 below for route identities). The travel time SD is generally higher for the inbound direction than the outbound direction (except for route 9, the Princess Highway between Haymarket and Arncliff, which is probably due to higher users travelling towards Sydney airport (via the CBD) and its neighbouring employment hubs). This observation makes sense as a majority of traffic moves towards the Sydney CBD during AM peak. This heavy movement of traffic often leads to congestion and occurrence of incidents which increases travel time. Appendix A5 presents a step-by-step procedure for applying the ARSD model on the Sydney case study.

Figure 4-16: Bidirectional travel time SD for arterial routes in Sydney

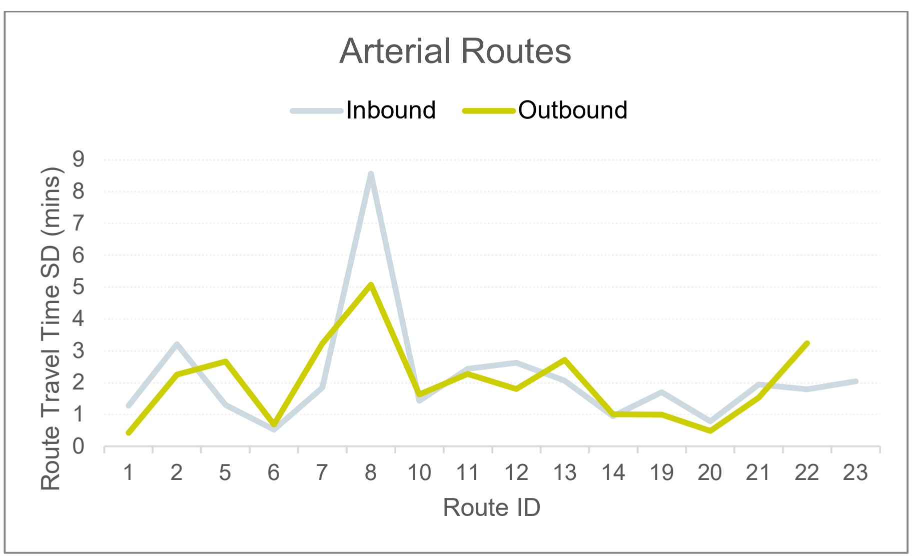
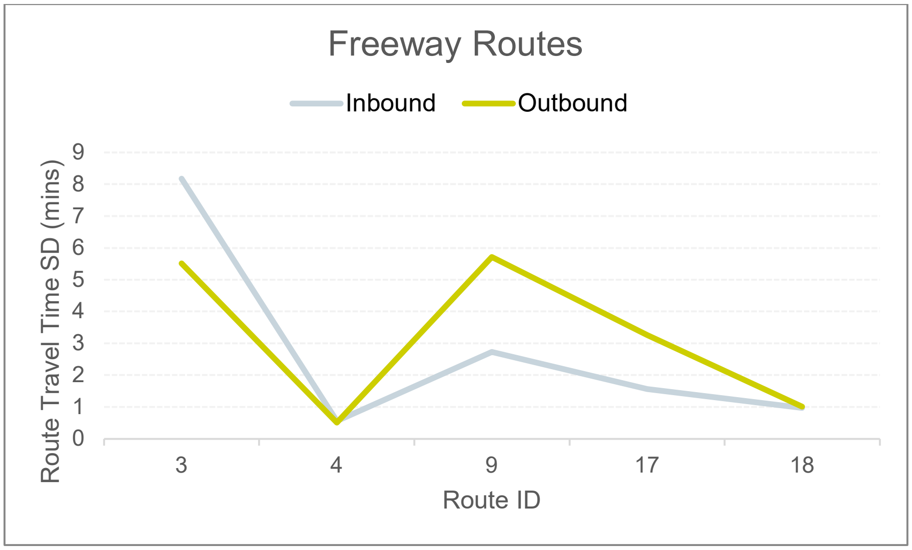


Figure 4-17: Bidirectional travel time SD for freeway routes in Sydney



4.3.3 Application of StrUE – Sydney Case Study

The following case study of Sydney, Australia presents the value of StrUE traffic assignment in network modelling applications, especially in the scenario where travel time reliability is an essential network output of the modelling context. This case study highlights how StrUE can capture the impact of changing network infrastructure on reliability, at link, route and network levels. It should be noted that the focus of the case study is to present the differing outputs and comparative studies which are possible using the StrUE framework. It does not provide extensive details regarding the network preparation, zoning, demand data collection, or base model calibration and validation as these are fundamental network modelling principles applicable to all forms of network modelling. Furthermore, the Sydney model has been developed using the data sources available within the project and a more comprehensive model can be developed with a more extensive data collection process.

Figure 4-18 represents the network map of Sydney, which consists of 143 zones, 10,540 links and 5,745 nodes. The following data sources were used to develop an 8AM to 9AM peak hour strategic model:

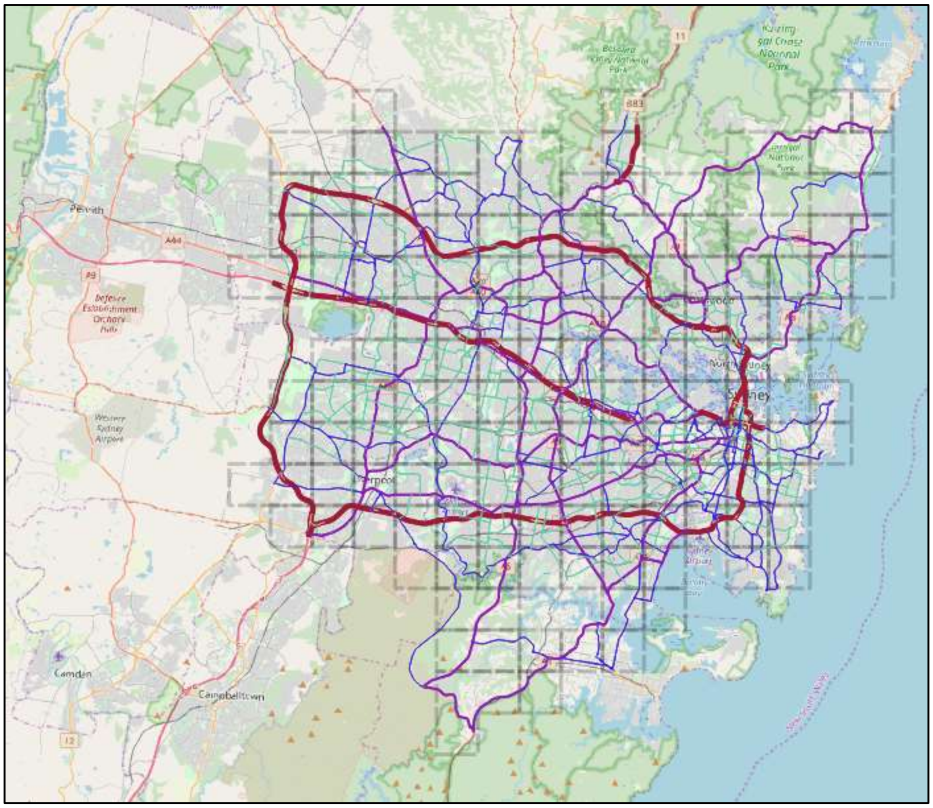
- **Calibration and Validation Data**: Travel time data for 69 routes in Sydney collected for a period of 6 months from April to September of 2018 using Google Maps API. The expected and standard deviations of travel time have been calculated during the defined morning peak hour of 8AM-9AM.
- **Demand Data**: The trip table for the network was estimated using a machine learning approach[12] such that the expected route travel times match with that of Google. Given that the OD matrix estimation is an underdetermined problem, multiple solutions for a trip table can exist that result in similar expected route travel times. Therefore, the developed matrix in the study may not be an accurate representation of real-world data[13] and needs further calibration and validation. However, as the purpose of this case study is to understand the application process, the feasible outputs and how they can be beneficial in measuring reliability as well as evaluating projects, this is not a critical component in this context. The project team has estimated the total expected demand of the network as 467,000 across a 1-hr peak period (8AM-9AM). However, the StrUE framework assumes a demand distribution and this cannot be obtained through traditional household travel surveys. Therefore, for this demonstration, it is assumed that the demand follows a lognormal distribution with a mean demand of 467,600. The lognormal distribution ensures that the demand is always positive, unlike a normal distribution.

---

[12] Methods like Random Forest, Convolutional Neural Network (CNN). Deep Neural Network (DNN) have been used in determining the real-time OD matrix (Chang and Edara (2017) and Ou et al. (2019)).

[13] The main purpose of the case study is to demonstrate the applicability of the StrUE model rather than the calibration.

Figure 4-18: Road network for Sydney case study



4.3.3.1 Calibration and Validation

Like other strategic models, there are numerous mechanisms to calibrate a network model using StrUE as a traffic assignment technique. The options include:

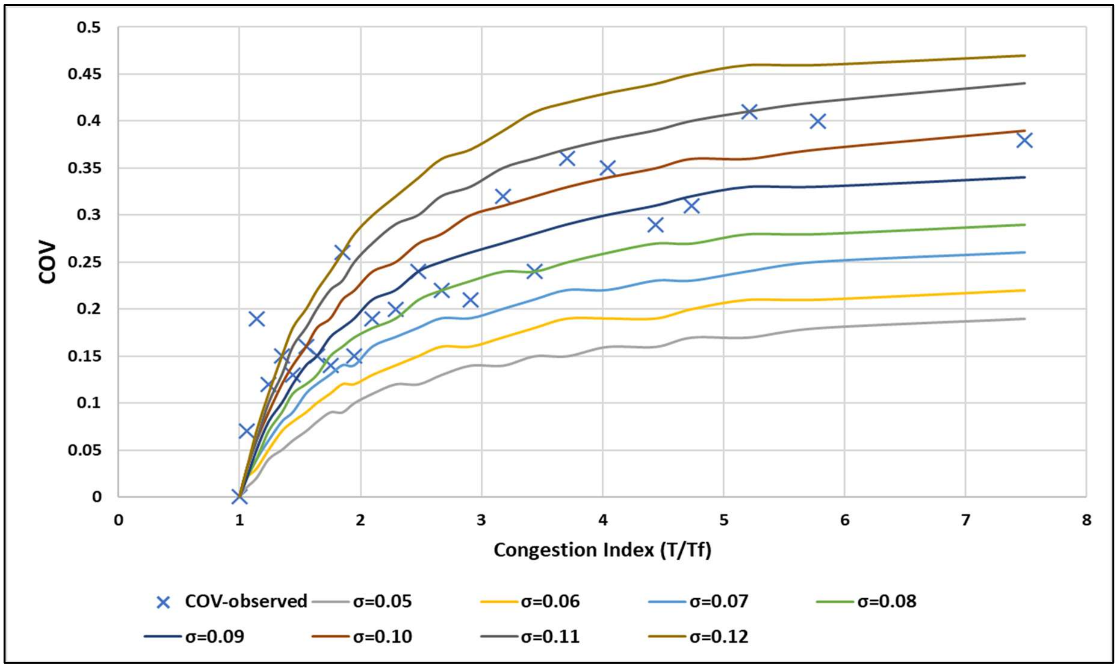
1. Comparing observed and modelled performance metrics (link volumes, link travel times and route travel times) using statistical approaches.
2. Trend analysis and distribution fitting to ensure that the relationship between two performance metrics are consistent between observed and modelled conditions.
3. Utilising Geoffrey E. Havers (GEH) metrics considering link volumes and travel times (Roads and Maritime Service Guide to Traffic Modelling, 2013).

The second option was selected to calibrate and validate the case study presented in this guideline. Different standard deviation of demand (σ) values are considered for the demand distribution. σ represents the spread of the distribution, which implies that an increase in this parameter widens the distribution of total demand. The "μ" parameter which denotes the mean of the distribution was held constant so that that the average demand stays at the value of 467,000.

Figure 4-19 presents the estimated relationship between CoV vs CI for various σ parameters of the StrUE model compared with the observed CoV vs CI values of the Google travel time data (points demarcated as X) of all the links along all the routes considered in the study. For a given CI, an increase in the σ parameter results in an increase in the coefficient of variation (CoV). Calibration involved comparing observed data to the modelled outputs.

To effectively discern trends in the large quantity of observed data and also to compare with modelled outputs, a data synthesis process was carried out using an "average-range" method to reduce noise. The method involved averaging CoV estimates for ranges of CI. Root-mean square error (RMSE) has been calculated between the observed (scatter points marked as X) and predicted CoV (trendline for a given σ) for different σ parameters, and the one with the lowest RMSE was chosen, that is σ = 0.10 as the best-fit to the observed data. Therefore, in the context of this case study, the model has been adequately calibrated.

Figure 4-19: Observed vs predicted relationship between CI and CoV of Google links



Note: The CoV-observed data are the averaged values for covariance ranges of all link data across all the routes this representation serves as a reflection of all the observed data

4.3.3.2 Scenario Testing

This section highlights both the reliability metric outputs that can be directly obtained from utilising the StrUE traffic assignment approach as well as the value of the approach in evaluating changes to the network from a reliability perspective. Several scenarios have been tested and the resulting outputs are shown in the sections below. The network changes considered can be summarised as:

- Scenario 1: Capacity of all links in the network increased by 10%
- Scenario 2: The speed limits on all links decreased by 10 kph
- Scenario 3: Capacity increase for a single link on the network.

The first two scenarios consider network wide changes, while the third scenario reflects an infrastructural upgrade project that is commonly undertaken by transport authorities. For example, it is comparable to adding a lane on a major arterial road. The assumptions made for the scenario testing are: (i) the average demand is 467,000, and (ii) SD of demand, σ is 0.10. The StrUE framework allows testing of the impact of network modifications on travel time reliability, thus facilitating in a before-versus-after comparison.

Scenario 1: Capacity of all links in the network increased by 10%

The impact of increasing capacity of all the links by 10% on expected travel time, standard deviation of travel time, and CoV of travel times on all links in the network is analysed. Expectedly, for almost all the links, these metrics have decreased when compared to the base scenario. Table 4-7 presents percentage changes of outputs for a randomly selected sample of 10 links in the network, demonstrating the direct capability of the modelling approach. While a majority of the links (8 out of 10) show a drop in CoV as a result of capacity increase, 2 out of 10 links show a positive change which indicates an increase in CoV upon this intervention. The justification for this observation is as follows: It was found for these two links that both mean and SD of travel time reduced as a result of the capacity increase, and both reductions are realistic. However, the improvement in travel time reliability (measured by SD of travel time) is not as significant as the reduction in mean travel time, and thus CoV increases. In summary, it can be observed that the increase in capacity had a positive effect on travel time savings as well as improved reliability.

Table 4-7: Scenario-1 - Percentage change in travel time metrics for randomly selected links in the network

Route	From Node Coordinates	To Node Coordinates	Suburb	Link Length (m)	% change in Expected TT	% change in SD of TT	% change in CoV of TT
M1 General Holmes Dr	-33.936771, 151.1978781	-33.9338887, 151.2106665	Mascot	1277	-11	-6	5
A1 Pacific Highway	-33.8354889, 151.2053255	-33.8276538, 151.2006783	North Sydney	981	-4	-10	-6
A34 Milperra Road	-33.9360905, 151.0109709	-33.9298806, 150.9908058	Revesby	1991	-12	-13	-1
A40 Botany Road	-33.921096, 151.1967991	-33.9055165, 151.2028326	Alexandria	1869	-17	-59	-50
A40 New South Head Road	-33.8709482, 151.251437	-33.8724953, 151.2598255	Bellevue Hill	885	-14	-7	9
A1 Pacific Highway	-33.8053918, 151.1794154	-33.8104226, 151.1771309	Artarmon	642	-9	-35	-29
A8 Pittwater Road	-33.7887083, 151.2652612	-33.7955396, 151.2530601	Balgowlah	1577	-5	-36	-32
A40 Victoria Road	-33.8166083, 151.1093996	-33.8160805, 151.1039224	Ryde	547	-7	-47	-43
A1 Princes Highway	-34.0238036, 151.0890585	-34.0266529, 151.0850233	Kareela	500	-3	-30	-28
A44 The Horsley Dr	-33.8886837, 150.9695904	-33.8846809, 150.9674909	Villawood	495	-15	-40	-29

However, the benefit of the StrUE approach is that these above output metrics can be extracted for the entire network, providing a more holistic and comprehensive assessment, infeasible with the standard User Equilibrium approach.

Figure 4-20, Figure 4-21 and Figure 4-22 show the percentage change in expected travel times, standard deviation, and coefficient of variation of travel times for routes from each zone to the CBD. The expected travel times decreased by 12% to 26%, whereas the standard deviations decreased by 7% to 41%, with most improvements seen in the North West region of Sydney (highlighted in dark green in Figure 4).

The change in expected travel times is more homogeneous than the standard deviation. While there is significant reduction in expected TT from the North Shore and Inner West regions of Sydney, the reduction in SD of TT is not as much when compared to other regions.

Figure 4-20: Scenario-1 - Percentage decrease in expected TT from different zones to CBD

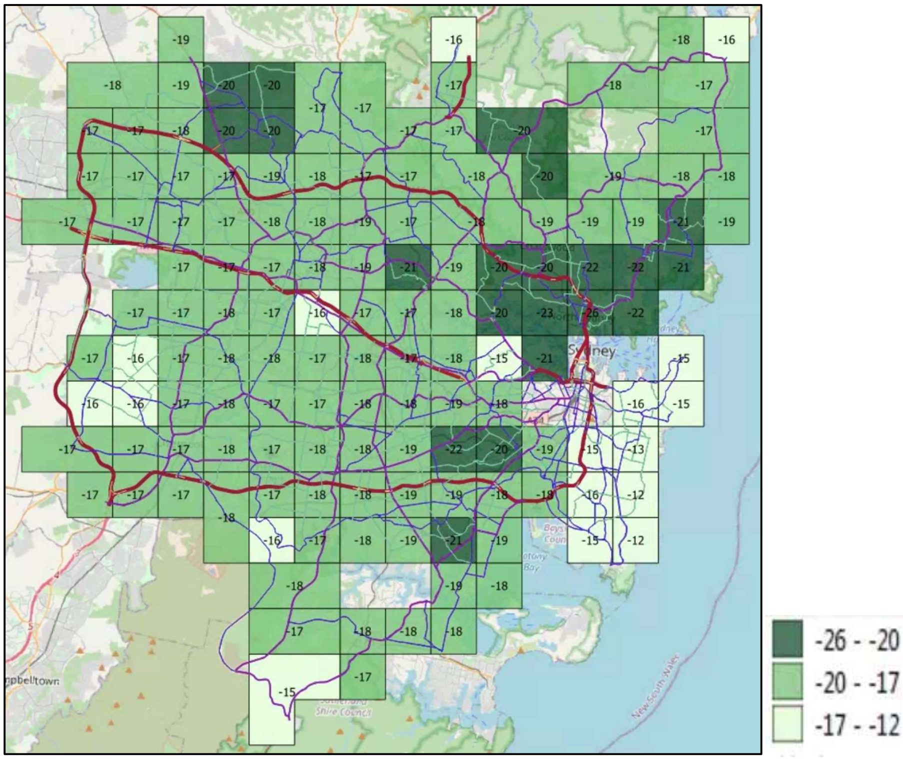


Figure 4-21: Percentage decrease in SD of TT from different zones to CBD

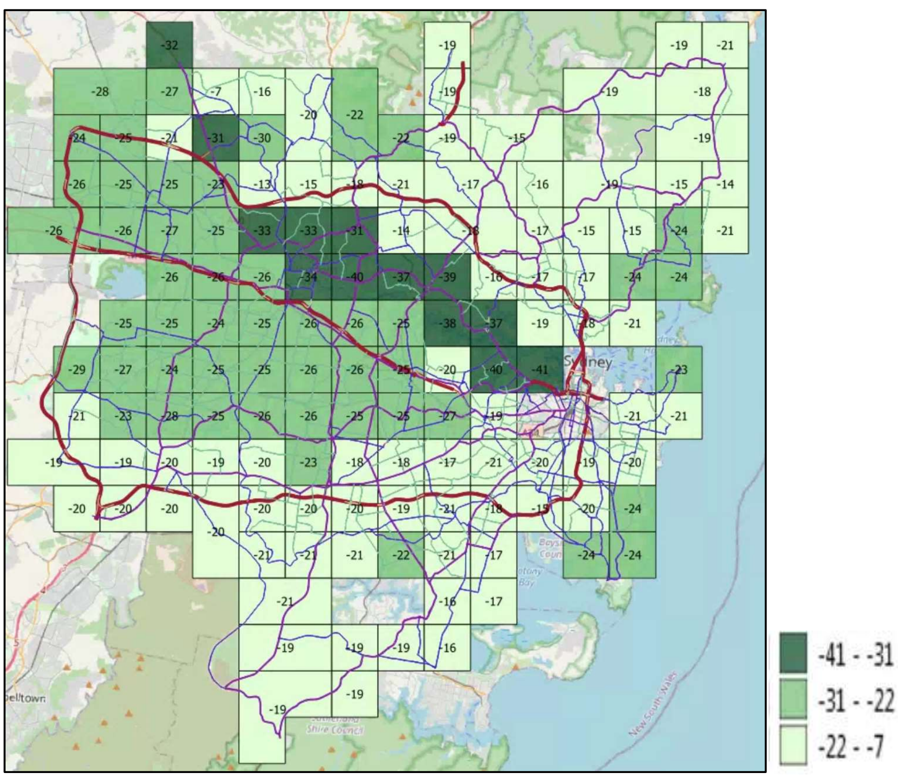
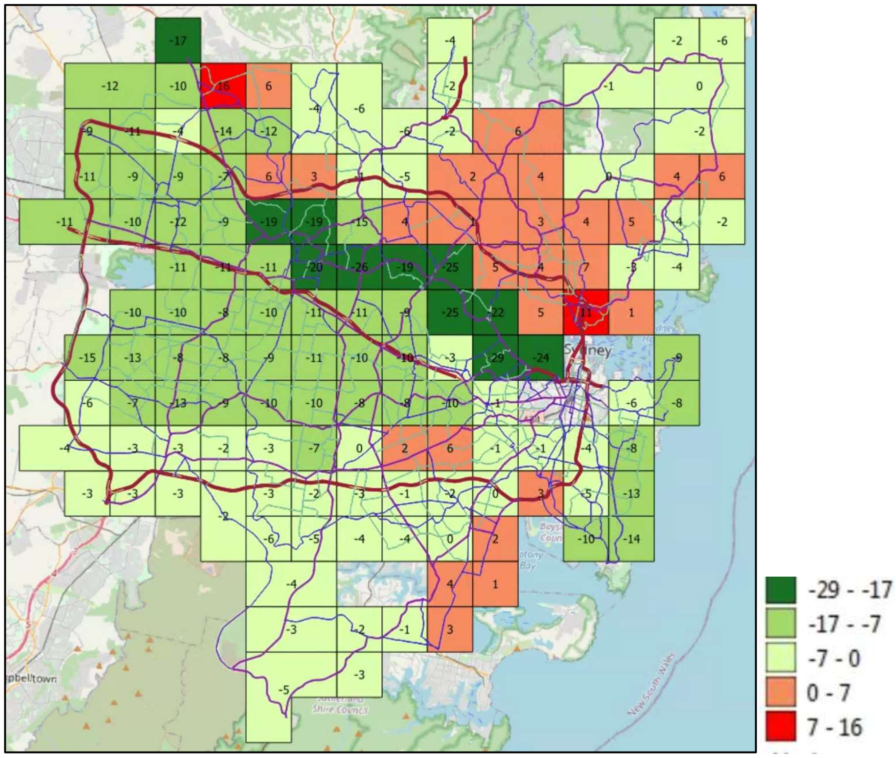


Figure 4-22: Scenario-1 - Percentage change in CoV from different zones to CBD



Note: -ve denotes reduction and +ve for increase.

Appendix A6 presents a detailed discussion on the other two scenarios. This methodology offers direct estimation of standard deviation of travel time for each link and route of the network, thus providing a clear path to measuring and monitoring reliability. The scenario analysis clearly indicates that policies and infrastructure projects do not have homogenous impacts across a network or even within a sub-network. This emphasises the importance of network analysis that endogenously incorporates reliability within the modelling framework.

4.3.4 Comparison between ARSD and StrUE

Comparing the network travel time variability models is important to guide practitioners on use cases for both approaches. The assumptions underlying each method dictate the applications for both models. As presented in the previous sections of the report each model serves the following purpose:

- The ARSD model is an empirical method to measure reliability at a route level using historical travel time data. The model exogenously accounts for reliability and does not capture route choice behaviour where travellers consider reliability in their decision-making process. Thus, the ARSD model is suitable to assess short term reliability impacts related to microscopic operational changes in the network.

- StrUE is a traffic assignment methodology that could be substituted as the final step of a traditional 4-step travel model (such as STM). The model endogenously accounts for reliability capturing the concept within the route choice behaviour across the network. Accordingly, this form of modelling is ideal to assess network wide reliability impacts related to both localised and network modifications of the system. This approach is currently the only mechanism in the world which translates reliability as a part of the travel decision making process, which is iterated until a reliability inclusive traffic equilibrium is developed. This means that reliability metrics such as standard deviation of travel time are direct outputs of the model, providing a robust foundation of sensitivity testing and “what-if” scenario analysis of major infrastructure changes.

It is critical to emphasise that ARSD is useful as an efficient method to estimate variability impacts for localised modifications in the network. However, it is limited in providing a reasonable quantification of reliability for significant network changes or macroscopic policy implementation as it does not consider travel behaviour within the model framework. One the other hand, the use of StrUE within a strategic model can provide robust results for localized and network-wide changes at a system level. Accordingly, this approach is the preferred option when route choice and network impacts are anticipated such as in major infrastructure projects. StrUE can then be used for network reliability assessments of major infrastructure projects.

5 Summary and Next Steps

5.1 Summary

Link travel time variability

Eleven different models to estimate link and route travel time variability were identified from the literature review. A numerical experiment was conducted comparing the three shortlisted models from the literature review along with an ‘ATAP’ model, of which the latter was found as a better fitting model. The ATAP link model was calibrated using the entire NetPReS dataset. Two separate ATAP models were calibrated, one each for the arterial and freeway data in the entire NetPReS dataset. The model calibration was undertaken using linear regression (for Equation 3.2) in MS-Excel and then converting back Ln(CoV) into CoV. The estimated parameters were found to be statistically significant at 95% confidence and the R-squared value was close to 0.56 for arterial and 0.67 for freeway. A visual validation of the two ATAP models was also undertaken on datasets from other jurisdictions (Gold Coast Bluetooth data and Sydney Google data) and both the models were found to have a reasonable goodness-of-fit. Thus, this project recommends the use of the ATAP link model, given in Equations 4.2 (that is $\boldsymbol{CoV} = \mathbf{0.5939}.\left(\frac{CI-1}{CI}\right)^{\mathbf{0.968}}\ \forall\ \boldsymbol{CI} \geq \mathbf{1}$) and 4.3 (that is $\boldsymbol{CoV} = \mathbf{0.7913}.\left(\frac{CI-1}{CI}\right)^{\mathbf{1.08}}\ \forall\ \boldsymbol{CI} \geq \mathbf{1}$), to determine link travel time variability nationally.

Some limitations of the developed ATAP model are as follows. Firstly, not considering the effect of other factors such as weather, incidents, events, number of roundabouts and bottlenecks, geometric conditions on CoV. Some of that information was not available in the data used under this project, others such as weather-related information did not showed noticeable impact on travel time variation. Secondly, the calibrated models might perform sub-par (with regard to the goodness-of-fit) on other jurisdictional data. This is mainly due to significant geographical differences in the traffic characteristics which could affect travel time variability in that jurisdiction. Nonetheless, it is expected that the model would perform reasonably well when applied on other jurisdictional data, with some variations attributed to the level of error in the model and local traffic characteristics.

Route travel time variability

Two methods of determining route travel time variability were identified in the literature, of which the SD based approach was selected due to its advantages. The CRM recommended by Nicholson (2015) expresses route travel time SD in terms of: (i) travel time SD of constituting links and (ii) the degree of correlation between these links. While the former can be determined using an appropriate ATAP model (for arterial/freeway), the latter can be modelled using CCM and the ATAP link model. Three different functional forms (exponential, shifted exponential and linear-log) for the CCM were tested on the entire NetPReS dataset. The entire dataset was segregated into 16 sub-samples based on road type (arterial/freeway), directionality (inbound/outbound) and time of the day (AM, inter, PM, off) as classifiers. The models were calibrated on each sub-sample. The exponential function (proposed in the literature) and the shifted exponential functional forms were found to have a poor goodness-of-fit statistics (e.g., R-squared) when compared to the linear-log form. The estimated parameters for the linear-log model, which were obtained using the linear regression technique, were also found to be statistically significant at 95% confidence. Thus, the linear-log form (that is, $\boldsymbol{\rho_{i,j}} = \boldsymbol{Max}[\mathbf{0}, \boldsymbol{a} \times \mathbf{Ln}(\mathbf{L}) + \mathbf{b}]$) has been recommended as the CCM for this project. The validation of the CRM was conducted using the route-level information available in the NetPReS dataset. The results from the validation were found to be satisfactory.

Some limitations of the CRM model are as follows. Firstly, the CCM only considers distance as the explanatory variable. It does not take into consideration other attributes due to lack of availability in the current NetPReS dataset, e.g., bottle-neck effects, which could potentially influence CCM. Secondly, the CRM has been calibrated and validated using the NetPReS data which might lead to poorer model fit for the data from other jurisdictions. Thus, practitioners need to re-calibrate the CRM using available data if required.

Network level travel time variability

For the network level travel time reliability modelling, this project explored two novel methodologies namely, the Approximate Route SD (ARSD) and the Strategic User Equilibrium (StrUE). The ARSD method applies a correction factor to the summation of individual links SD to obtain route travel time SD. Another numerical experiment was developed to assess the impact of the correction factor in the ARSD method. The correction factor value was then calibrated using the arterial and freeway routes in the full NetPReS dataset. The ARSD approach was applied to the Sydney case study to determine travel time variability on a few selected routes. Similarly, the application of StrUE model was also developed on the same Sydney case study to assess the impact of route travel time reliability in network assignment. StrUE is able to evaluate network wide reliability impacts, as it endogenously considers the impact of variability on route-choice and vice-versa. While the ARSD is an efficient method to estimate variability impacts for localised modifications in the network, it does not take into consideration its impact on route choice. On the other hand, StrUE, although more complex to develop than ARSD, is a more methodologically robust approach which endogenously takes into consideration travel time variability and its impact on route choice.

5.2 Limitations of the Work

This project has presented a new calibrated formulae and recommended methodologies for the practitioners to estimate travel time variability, and hence travel time reliability, in a consistent and principled way. As with any model development process, each phase of development is limited by scope, budget and timing. There is always room for further development and enhancements, especially in the context of travel time reliability where a general model would be expected to require further calibration to the local network, as road networks are subtly different, regionally and locally.

The limitations of this project at the present time are noted as follows:

- The link model (ATAP model) only considers the CI as an explanatory variable for modelling. The model did not take into consideration the effect of other attributes on travel time variability due to no noticeable impact on travel time reliability such as weather or traffic incidents and due to the lack of relevant information in the analysed datasets (discussed in sub-section 2.1.3). Similarly, the CCM model considers only link length as the explanatory variable. The extent of this limitation is low to medium which can be justified as follows: The ATAP link model has an RMSE value of around 0.1 on the NetPReS dataset which signifies an acceptable goodness-of-fit. While adding more explanatory variables will certainly lower this RMSE further, the extent is not expected to be significant. A similar justification can be made for the CCM which also indicates lower RMSE values (in the range of 0.1 and 0.2).
- For the CRM, instantaneous link travel times are considered to determine the route travel time. For example, the prevailing travel times for all links in a route at 9:00 am are added to determine route travel time. The model does not take into consideration the prevailing travel time from the actual time of arrival on a link, which is a more physically correct way to measure route travel time. The extent of this limitation is low. Even though the past research shows difference between instantaneous and experienced travel times which could impact the goodness of the CRM (Chiu et al., 2011), the assumption was found to be accurate to within 4% on average (Moylan et al., 2018).
- The CRM is composed of two sub-models, the ATAP link model and the CCM, and the unobserved errors associated with each tend to accumulate while estimating route travel time variability using the CRM. Attention to calibration with good quality local datasets will improve the CRM. The extent of this limitation is low which can be justified as follows: The RMSE heatmaps for CRM (Figure 4-11 and Figure 4-12) and the frequency distribution of the RMSE values (Figure 4-13) indicate that a majority of the values are around 0.5 which corresponds to an acceptable fit. Minimising unobserved errors by introducing more explanatory variables each in the ATAP link model and the CCM will further decrease the average RMSE value, but the scale is not expected to be significant.
- The calibration parameter γ value used in the ARSD method (Equation 3.4) has been calibrated using the NetPReS data. The parameter γ value would be improved with local calibration. Thus, practitioners are advised to re-calibrate the ARSD formula using their available data, if required. The extent of this limitation is medium as every jurisdiction has its own traffic characteristics and dynamics, and using the calibrated γ value given in this report might lead to greater errors in the method, that is ARSD, which in itself is an approximation to begin with.

Appendices

List of Figures in Appendices

List of Tables in Appendices

Appendix A

A1 Literature Review of Link and Route Models

This appendix provides a detailed discussion on the 11 link and route level models, for forecasting SD of travel time, that have been listed in Table 2-3 of the main report.

A1.1 UK Model (UKM)

The UK Model (UKM) was first developed and estimated by Arup (2003) using London and Leeds data collected in 1993 and 2003. The UK model was also estimated using Australia and New Zealand Data. The data used to develop models for the Australian context was collected through Google Maps. Equation A.1 provides the expression for the UKM, while Table A-1 provides the associated parameter values.

$$\boldsymbol{CoV} = \boldsymbol{a}\left(\frac{\boldsymbol{T}}{\boldsymbol{T_f}}\right)^{\boldsymbol{b}} \boldsymbol{D}^{\boldsymbol{c}} \qquad \text{[EQ A.1]}$$

Where:

CoV = coefficient of variation, $\frac{\sigma}{T}$

σ = SD of travel time (in s)

T = mean travel time (in s)

T_f = free flow travel time (in s)

D = length of link (in m)

$\mathrm{a, b, c}$ = parameters

Table A-1: Calibrated parameter values for the UKM

Source	Location	a (Constant)	b (T/T_f)	c (Length)
UK Transport Guidelines	UK	0.16**	1.02**	-0.39**
Osterle et al. (2017)	Adelaide	0.034**	6.54**	-0.04*
	Auckland	0.085**	2.97**	-0.09**
	Brisbane	0.028**	4.34**	0.01
	Canberra	0.032**	4.79**	-0.01
	Darwin	0.030**	8.74**	-0.09
	Hobart	0.038**	5.17**	-0.05
	Melbourne	0.060**	3.22**	-0.03
	Perth	0.051**	6.19**	-0.10**
	Sydney	0.117**	2.47**	-0.08**
	Wellington	0.028**	5.48**	-0.05
	All cities	0.044***	3.96***	-0.03

*Note: asterisks denote statistical significance: * at 10%, ** at 5%, *** at 1%.*

A1.2 Log-linear Model (LLM)

A log-linear model (LLM) is an alternative formulation to the UKM model. It is a different expression of the UKM but transformed by natural logarithms of the Equation A.1. While both the UKM and the LLM are inter-convertible, the LLM has been considered in the past studies, and hence included in this review. The model and parameters are calibrated for key roads in Australian and New Zealand. A stratification of the model based on approximate speed zone is also proposed. Note that the adoption of this model would require the results to be converted into minutes rather than seconds. Equation A.2 provides the expression for the LLM, while Table A-2 provides the associated parameter values.

$$\boldsymbol{Ln(CoV) = Ln(a) + b \times Ln\left(\frac{T}{T_f}\right) + c \times Ln(D)} \qquad \text{[EQ A.2]}$$

Where:

CoV = coefficient of variation, $\frac{\sigma}{T}$

σ = SD of travel time (in min)

$\frac{T}{T_f}$ = congestion index

T = mean travel time (in min)

T_f = free flow travel time (in min)

D = length of link (in m)

$\mathrm{a, b, c}$ = parameters

Table A-2: Calibrated parameter values for the LLM

Source	Location	Ln(a) (Constant)	b Ln (T/T_f)	c Ln(Length)
Osterle et al. (2017)	Adelaide	-3.374**	6.541**	-0.0408*
	Auckland	-2.466**	2.974**	-0.0923**
	Brisbane	-3.562**	4.342**	0.0108
	Canberra	-3.434**	4.793**	-0.00721
	Darwin	-3.52**	8.742**	-0.086
	Hobart	-3.283**	5.17**	-0.0499
	Melbourne	-2.814**	3.221**	-0.0318
	Perth	-2.98**	6.194**	-0.103**
	Sydney	-2.144**	2.466**	-0.0816**
	Wellington	-3.587**	5.477**	-0.0483
	All Cities	-3.128***	3.959***	-0.0328

*Note: asterisks denote statistical significance: * at 10%, ** at 5%, *** at 1%.*

A1.3 New Zealand Model (NZM)

The New Zealand Model (NZM) is also called the travel time variability model. The purpose of the model is to estimate travel time variability on a single link. This approach is fundamentally based on a mathematical relationship between the level of congestion (measured in V/C ratio) and the SD of travel time. It allows the estimation of the variability on individual road links using information on the volume and capacity of that link. A limitation of the model is that any additional travel time variability due to major incidents (crashes or breakdowns) are not included in the analysis which must be estimated separately. Equation A.3 provides the expression for the NZM, while Table A-3 provides the associated parameter values.

$$\boldsymbol{\sigma} = \boldsymbol{\sigma_0} + \frac{\boldsymbol{\sigma_1} - \boldsymbol{\sigma_0}}{\boldsymbol{1} + \boldsymbol{e}^{\boldsymbol{b}\left(\frac{\boldsymbol{V}}{\boldsymbol{C}} - \boldsymbol{a}\right)}} \qquad \text{[EQ A.3]}$$

Where:

$\sigma =$ SD of travel time (in min)

$\sigma_0 =$ lower limit of SD (in min)

$\sigma_1 =$ upper limit of SD (in min)

$\frac{V}{C} =$ volume (demand) to capacity ratio

$a, b =$ parameters

Table A-3: Calibrated parameter values for the NZM

Source	Road stereotype	σ_0	σ_1	a	b
New Zealand Economic Manual as cited in Moylan et al. (2018)	Motorway	0.083	0.90	1	-52
	Urban arterial	0.117	0.89	1	-28
	Urban retail	0.150	0.87	1	-16
	Urban other	0.050	1.17	1	-19
	Rural	0.033	1.03	1	-22
Moylan et al. (2018)	Sydney motorway	0.040	2.07	1	-52
	Sydney arterial	0.092	5.22	1	-28

From Tables A-1 to A-3, the parameters adopted in the UKM and the NZM varied among the countries and cities. These variations may be due to the geographical characteristics of their transport network configuration, land use planning and the provision of public transport. As the operation of toll routes is beyond the scope of this report, all routes/links studied are open to free public access.

A1.4 Unified Reliability Model (URM)

Moylan et al. (2018) proposed the unified reliability model (URM). The URM model is an adaptation of the UKM. It aims to overcome the limitations of other models that tend to focus on travel time, trip length and traffic flow as the key independent input variables that affect reliability. The URM aims to capture additional factors like temporal impacts including but not limited to time of the day, proximity to urban centres and road capacity limitations of the network. It also tries to overcome the issue of variation of time-periods, infrastructure types and location which will typically require outputs from multiple different models. The development of the final URM model requires interim modelling steps. The final URM model removes any collinearity between the variables which reduces the equation to the form that only includes significant variables. As stated in the report, no local streets were involved in model comparison, calibration and validation. Typically, the model had an adjusted R^2 of 0.59 for the data indicating a decent goodness-of-fit of the model. Equation A.4 provides the expression for the URM, while Table A-4 provides the associated parameter values.

$$\boldsymbol{\sigma = KT^{a}D^{b}} \qquad \text{[EQ A.4]}$$

Where:

σ = SD of travel time (in s)

$K = e^{c+dC+eP+f}$

C = capacity in pceu/h/ln $\begin{cases} 2{,}200 \; for\ motorway \\ 1{,}800 \; for\ arterial \end{cases}$

P = 1 if peak period, 0 otherwise

G = route type $\begin{cases} 0 \; for\ inner\ ring\ routes \\ 1 \; for\ middle\ ring\ routes \\ 2 \; for\ outer\ ring\ routes \end{cases}$

T = mean travel time (in s)

D = link length (in km)

a, b, c, d, e, f = parameters

Table A-4: Calibrated parameter values for the URM

	a	b	c	d	e	f
Related factor	Mean travel time	Link length	Constant	Capacity	Peak period	Route type
Parameter value	2.8496	-1.8875	-19.0339	0.0044	0.3923	0.2517

Source: Moylan et al. (2018)

A1.5 Linear Model (LM)

The linear model (LM) is a simple linear equation that relates the mean travel time to the SD. The linear model was developed by Hellinger (2011) as cited in Kouwenhoven and Warffemius (2017). Travel times were derived from detection loop data, averaged over 15-minute periods. Each 15-minute period represented one data point. It excluded trips after 11PM. The final dataset consisted of 92 points. The model was found to have an adjusted R2 of 0.75 for the morning peak period. The study concluded that a linear relationship was sufficient for shorter routes. There were limitations associated with the model for longer routes as a decreasing slope was observed on the longer routes under traffic congestion (Kouwenhoven and Warffemius, 2017). Equation A.5 provides the expression for the LM, while Table A-5 provides the associated parameter values

$$\boldsymbol{\sigma = a + bT} \quad \text{[EQ A.5]}$$

Where:

σ = SD of travel time (in min)

T = mean travel time (in min)

a, b =parameters

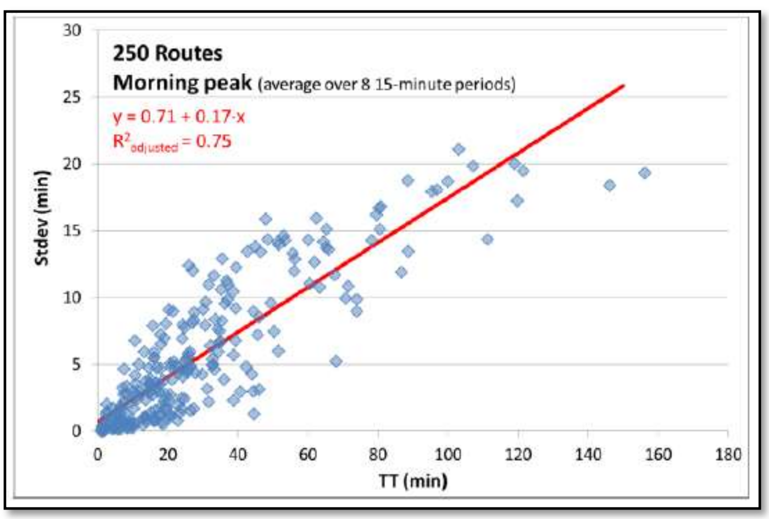
Table A-5: Calibrated parameter values for the LM

	a	b
Parameter value	0.71	0.17

Source: Kouwenhoven and Warffemius (2017)

Figure A-1 shows a plot for the morning period for which the LM has the best model fit.

Figure A-1: Linear model (LM) for morning peak and the line of best fit



A1.6 Length Standardised Linear Model (LSLM)

This length standardised linear model (LSLM) was developed as part of the Strategic Highway Research Programme (SHRP2) by Mahmassani et al. (2014), as cited by Kouwenhoven and Warffemius (2017). It models the SD of travel time for different road lengths. This network model approach has an advantage that it can also be applied if multiple routes are used between A and B with different lengths. It is especially suitable for dense urban networks. The mode which relates travel time per unit length and SD per unit length was found to depict a high goodness-of-fit value ($R^2 = 0.78$). Equation A.6 provides the expression for the LSLM, while Table A-6 provides the associated parameter values.

$$\frac{\boldsymbol{\sigma}}{\boldsymbol{L}} = \boldsymbol{a} + \boldsymbol{b}\frac{\boldsymbol{T}}{\boldsymbol{L}} \qquad \text{[EQ A.6]}$$

Where:

σ = SD of travel time (in min)

T = mean travel time (in min)

L = length (in km)

a, b = parameters

Table A-6: Calibrated parameter values for the LSLM

	a	b
Parameter value	-0.41	0.78

Source: Kouwenhoven and Warffemius (2017)

A1.7 Length Standardised Cubic Model (LSCM)

The length standardised cubic model (LSCM) was developed by Mott McDonald in the UK (cited by Kouwenhoven and Warffemius (2017)). The LSCM forecasts day-to-day changes in travel time after accounting for all predictable variations (time of day effects, day type effects and seasonal effects) and variability due to incidents. The data was collected by inductive loop sensors, automatic number plate recognition and matching and GPS tracking averaged over 15-minute periods on several highway routes. Incidents were controlled for by removing data points that were 2 SD above the mean. Mean journey time per kilometre versus SD per kilometre for several motorway types was also presented. They presented graphs with mean journey time per kilometre versus SD per kilometre for several motorway types. The SD of travel time per kilometre was presented as a cubic polynomial of the mean travel time per kilometre. The conclusions of the Kouwenhoven and Warffemius report (2017) was that a linear function was sufficient for expressing the SD per kilometre as a function of the travel time per kilometre, and that applying a cubic polygon does not improve the fit. Equation A.7 provides the expression for the LSCM, while Table A-7 provides the associated parameter values.

$$\frac{\boldsymbol{\sigma}}{\boldsymbol{L}} = \boldsymbol{a} + \boldsymbol{b}\frac{\boldsymbol{T}}{\boldsymbol{L}} + \boldsymbol{c}\left(\frac{\boldsymbol{T}}{\boldsymbol{L}}\right)^2 + \boldsymbol{d}\left(\frac{\boldsymbol{T}}{\boldsymbol{L}}\right)^3 \qquad \text{[EQ A.7]}$$

Where:

σ = SD of travel time (in min)

T = mean travel time (in min)

L = length (in km)

a, b, c, d = parameters

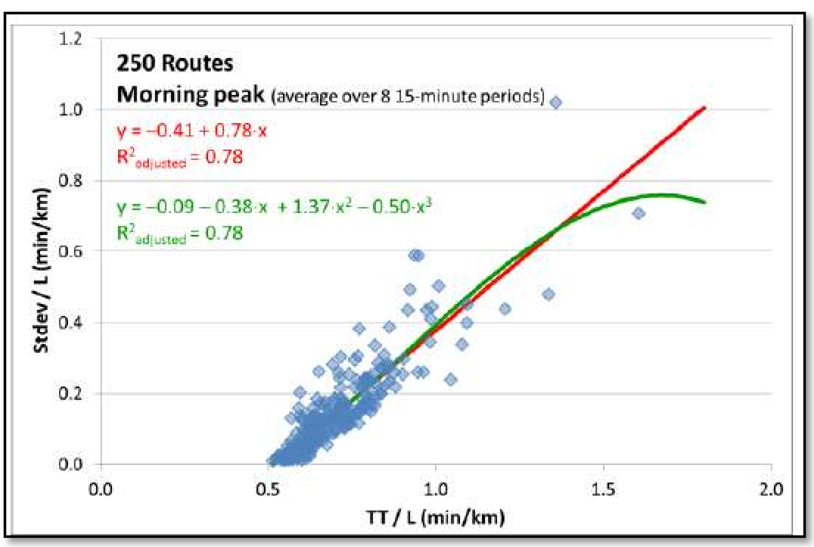
Table A-7: Calibrated parameter values for the LSCM

	a	b	c	d
Parameter value	-0.09	-0.38	1.37	-0.5

Source: Kouwenhoven and Warffemius (2017)

Figure A-2 compares the two model fit lines: 1) LSLM (red) vs. LSCM (green). Both have an R^2 of 0.78.

Figure A-2: Travel time per km vs. SD per km. Comparison of LSLM (Red) against LSCM (Green)



A1.8 Exponential Coefficient of Variation Model (ECVM)

The exponential CoV model (ECVM) model was developed by Eliasson (2006) and cited by Kouwenhoven and Warffemius (2017). Eliasson (2006) fitted an exponential function to the CoV for 20 roads and for 96 fifteen-minute periods in Stockholm, Sweden. The road lengths analysed varied between 300m and 5km. Equation A.8 provides the expression for the ECVM, while Table A-8 provides the associated parameter values.

$$\boldsymbol{CoV = exp\left(a + b\left(\frac{T}{T_f} - 1\right) + c\left(\frac{T}{T_f} - 1\right)^3\right)} \qquad \text{[EQ A.8]}$$

Where:

CoV = coefficient of variation, $\frac{\sigma}{T}$

σ = SD of travel time (in min)

T = mean travel time (in min)

T_f = free flow travel time (in min)

a, b, c = parameters

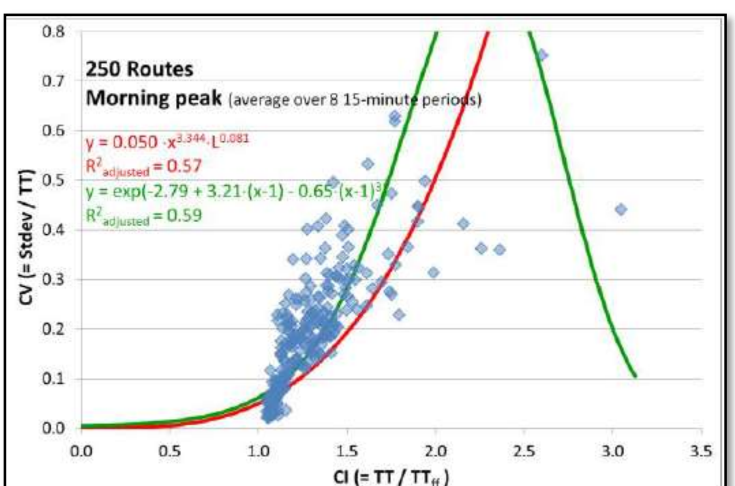
Table A-8: Calibrated parameter values for the ECVM

	a	b	c
Parameter value	-2.79	3.21	-0.65

Source: Kouwenhoven and Warffemius (2017)

The model indicated that the CoV remained roughly constant for lower levels of congestion and increased for slightly higher levels. At high levels of congestion, the CoV decreased again. Figure A-3 provides the CI (x-axis) vs. the CoV (y-axis) for the two models: 1) The red being a power law model (not evaluated in this paper), and 2) the exponential function (green line). From the data it was observed that low congestion levels did not appear to have a roughly constant CoV. A justification for this could be that the data included more of longer highway routes rather than shorter urban routes (Kouwenhoven and Warffemius, 2017). There were also insufficient data points at high congestion levels, so it may be difficult to establish if the data aligns with the predicted model behaviour.

Figure A-3: CI vs. CoV for 250 routes. Exponential function in Green



A1.9 Power Mean Delay Model (PMDM-1)

The power mean delay model (PWDM-1) was developed by Geistefeldt et al. (2014) and cited by Kouwenhoven and Warffemius (2017). The aim was to develop a reliability of travel time on their highways. The coefficients were simulated from a macroscopic traffic simulation model. The suggested model used a power-law function between the SD and the mean delay (that is the difference between the mean travel time and free flow travel time). The model had a goodness of fit of R^2 = 0.82. The mean delay as an explanatory variable may be appropriate for model development. Equation A.9 provides the expression for the PMDM-1, while Table A-9 provides the associated parameter values.

$$\boldsymbol{\sigma = aD^b} \qquad \text{[EQ A.9]}$$

Where:

σ = SD of travel time (in min)

D = mean delay (in min)

a, b = parameters

Table A-9: Calibrated parameter values for the PMDM-1

	a	b
Parameter value	1.63	0.73

Source: Kouwenhoven and Warffemius (2017)

Figure A-4 on the following page presents the PMDM-1 model. The PMDM-1 model is presented alongside another model, the Polynomial Mean Delay Model (PMDM-2) which is presented next.

A1.10 Polynomial Mean Delay Model (PMDM-2)

The polynomial mean delay model (PMDM-2) was developed by Peer et. Al (2012) and cited in Kouwenhoven and Warffemius (2017). It is an estimation between the SD and the mean delay. In principle, the mean delay is the difference between the observed and free flow travel times. The authors tried multiple functions on data from 145 highway routes across 57 fifteen-minute periods. The recommended function included, but were not limited to, a cubic polynomial in the mean delay and a quadratic polynomial in the length. The R2 of the model was approximately 0.96 which is a very good fit to the data. Equation A.10 provides the expression for the PMDM-2, while Table A-10 provides the associated parameter values.

$$\boldsymbol{\sigma = a + bD + cD^2 + dD^3 + eL + fL^2} \quad \text{[EQ A.10]}$$

Where:

σ = SD of travel time (in min)

D = mean delay (in min)

L = length (in km)

a, b, c, d, e, f = parameters

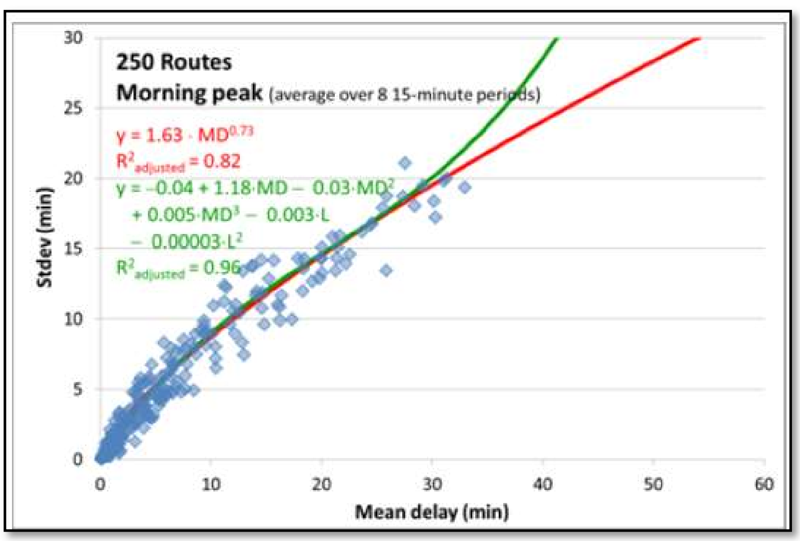
Table A-10: Calibrated parameter values for the PMDM-2

	a	B	c	D	e	f
Parameter value	-0.04	1.18	-0.03	0.005	-0.003	-0.00003

Source: Kouwenhoven and Warffemius (2017)

Figure A-4 presents the plot in which the green curve is the PMDM-2 while the red curve is PMDM-1.

Figure A-4: Plot comparing PMDM-1 (Red) against PMDM-2 (Green)



A1.11 Dutch Model (DM)

The Dutch model (DM) was developed by Kouwenhoven and Warffemius (2017) following a review of many of the previous models. Their conclusions included that the best empirical relationship to describe reliability was an expression of the SD as a function of the mean delay and length of the route. Other functional forms that were reviewed had a much lower adjusted R^2 or showed a behaviour that was not supported by the data. The Dutch model was a combination of a linear and logarithmic function for the mean delay and added a linear term in the length. Higher order terms and the terms proportional to other parameters such as density, number of lanes, average weather conditions, and frequency of incidents, were not found to be significant. The data was also delineated into morning peak, mid-day and afternoon peak. Equation A.11 provides the expression for the DM, while Table A-11 provides the associated parameter values.

$$\boldsymbol{\sigma = a + bD + c\,log_{10}(D+1) + dL} \qquad \text{[EQ A.11]}$$

Where:

σ = SD of travel time (in min)

D = mean delay (in min)

L = length (in km)

a, b, c, d = parameters

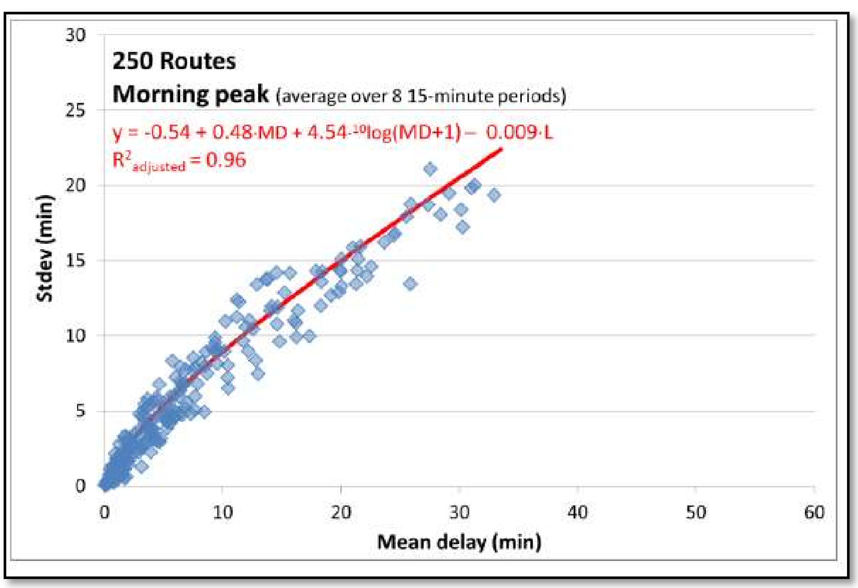
Table A-11: Calibrated parameter values for the DM

	a	b	c	D
Parameter value	-0.54	0.48	4.54	-0.009

Source: Kouwenhoven and Warffemius (2017)

Figure A-5 presents the DM fitted on the morning peak data. The model had an R^2 = 0.96 which was much higher than the previously evaluated models.

Figure A-5: The DM fitted to morning peak data



A2 Numerical Experiments

This appendix presents three numerical experiments which were conducted under this project. The first two experiments correspond to the link models, one comparing all reviewed (eleven) models and the other comparing only the shortlisted (four) models. The third numerical experiment is conducted to test the ARSD method at a network level.

A2.1 Comparing Reviewed Models

Equation A.12 shows the Bureau of Public Roads (BPR) function used to compute link travel time using a given V/C ratio.

$$\boldsymbol{T = T_f\left(1 + a\left(\frac{V}{C}\right)^b\right)} \quad \text{[EQ A.12]}$$

Where:

T = mean travel time (in min)

T_f = free flow travel time (in min)

V = volume (in pceu/h)

C = capacity (in pceu/h)

a, b = parameters ($a = 0.474$, $b = 4$)

Figures A-6 and A-7 show the travel speed and CI as a function of BPR for a given free flow speed.

Figure A-6: Speed versus V/C ratio, BPR function with free flow speed of 80 km/h

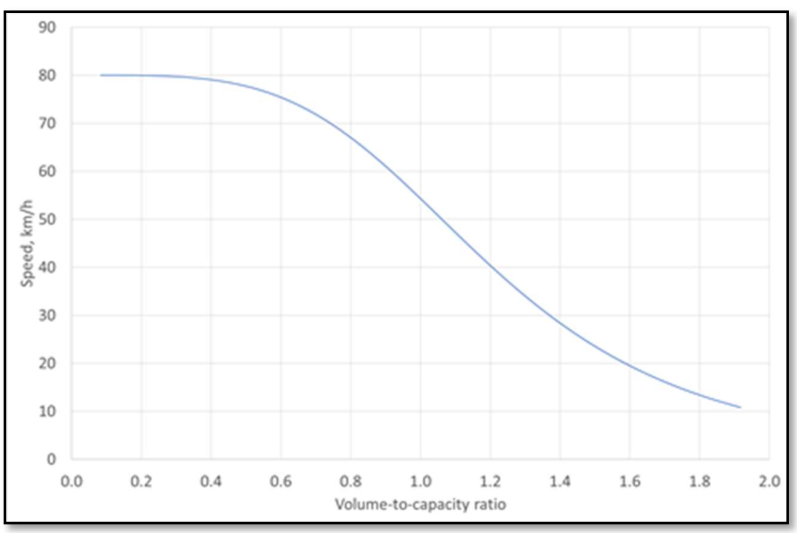
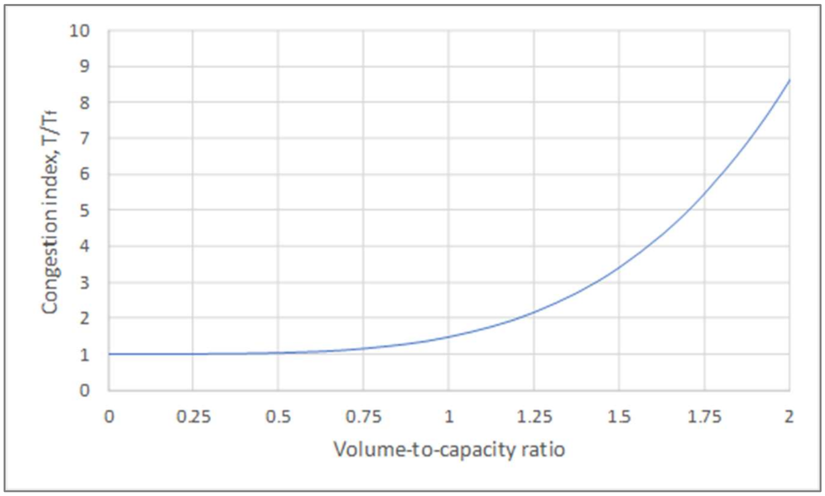


Figure A-7: Congestion index versus V/C ratio, BPR function with free flow speed of 80 km/h



Route based measure is the most logical way to assess travel time reliability. In some cases, particularly for arterial roads, a route may contain a few links which are in different road classes. As a result, the mixed speeds and road capacities may introduce more errors into models. The use of 1 km length in the numerical experiments was to test the performance of models under a similar situation (On a similar link, the T/T_f will be approximately the same).

A2.1.1 Volume (Demand) to Capacity (V/C) Approach

The ability to compare the models is an important part of this literature review, as it enables examination of the models' attributes to reasonably predict the CoV and SD of travel time based on V/C ratio and a CI, and test the likely relationship between the CoV and SD of travel time and link length.

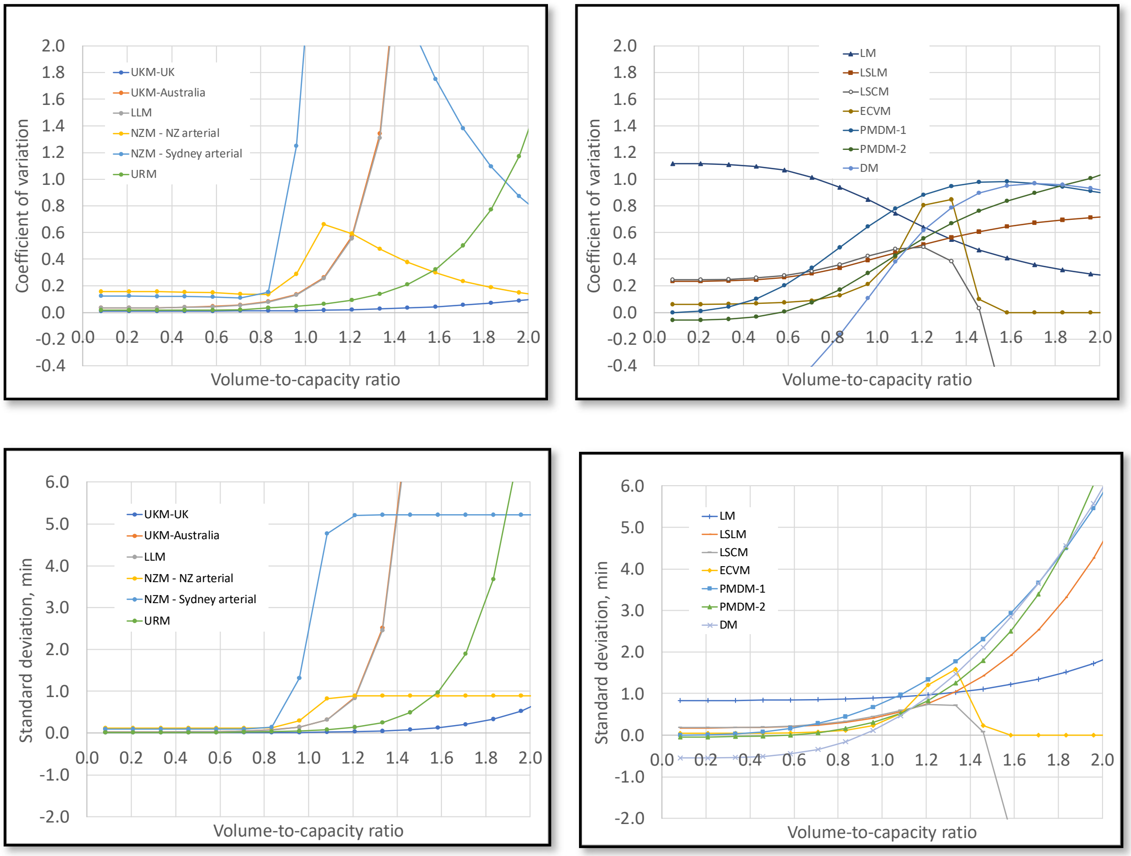
The results from the numerical experiment are as follows:

- CoV and SD of travel time under various congestion levels are shown in Figure A-8 where the independent variable is the V/C ratio with volume equal to demand and Figure A-9 where the independent variable is the CI, all cases assuming link length is 1 km.
- CoV and SD of travel time under various link lengths up to 2 km are shown in Figure A-10 (assuming a V/C ratio of 1.2).

There are eleven models that we have considered but the figures present a total of 13 results as there are two UKM and NZM versions. These being the UKM – UK version and UKM – Australia version and the NZM – NZ arterial version and NZM – Sydney arterial version. The results are presented in two charts each, the chart on the left with six of the models and the chart of the right with the remaining 7 models.

In Figure A-8 below, the first two charts on the top half of the figure show the relationship between CoV and V/C. Similarly, the two charts on the bottom half show the relationship between SD of travel time (in minutes) and V/C.

Figure A-8: CoV and SD of travel time versus V/C ratio on arterial highway – existing models



The desired travel time reliability model needs to be capable of correctly predicting both CoV and SD of travel time based on the required input data. From the eleven models reviewed, three models focused on estimation of CoV. Eight models focused on estimation of SD of travel time. The values of CoV and SD of travel time are interchangeable by using formula: CoV = SD of travel time / mean travel time. The relationship of CoV and SD of travel time with V/C ratio are plotted for all the eleven models in Figure A-8.

The predicted value of travel time reliability or SD of travel time by those 11 models are consistent for V/C ratio below 1, except LM, and DM. For V/C ratio above 1, the trends predicted by those models alter significantly. Most models predict the value for SD of travel time will increase as V/C ratio increased e.g. UKM, LLM, URM, LM, LSLM, PMDM-2 and DM, NZM predicts a constant SD of travel time when the V/C ratio reaches a certain point (V/C = 1.1 or 1.2), while others models namely LSCM and ECVM predict the value for SD of travel time will decrease as V/C ratio increased.

For travel time CoV graphs, the conflict between trends predicted by different models for V/C ratio greater than 1 is also observed. However, a majority of the 11 models predict the value of CoV to decrease as V/C ratio increases, those models are NZM, LM, LSCM, ECVM, PMDM-1, PMDM-2 and DM. The cause of the decline in the CoV in those models were examined. The explanation is that the decline is due to the difference in the rate of increase for SD of travel time and travel time when V/C ratio increases. Using the NZM as an example, when the V/C ratio reaches a certain point (V/C = 1.1 or 1.2), the SD became constant. However, the travel time continues to increase thus causing a divergence in the CoV chart.

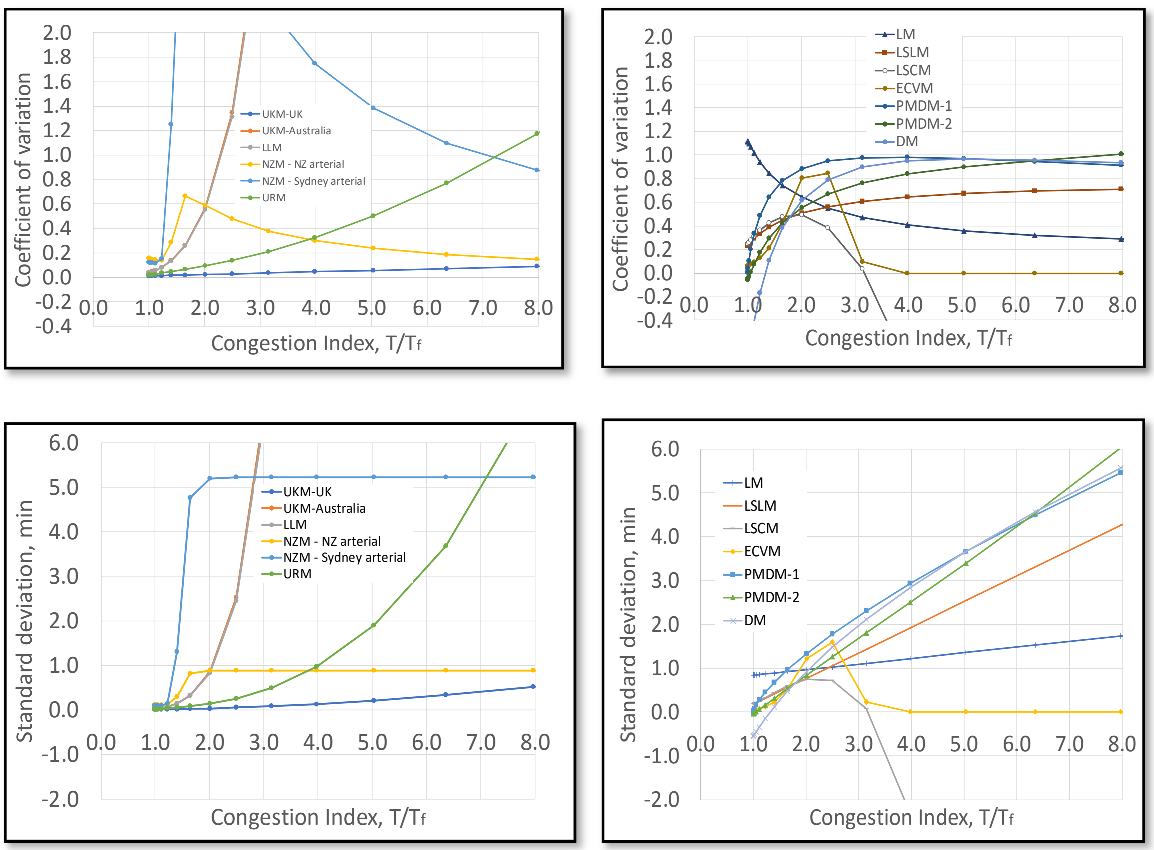
On the left-hand side of Figure A-8, the UKM, URM and LLM models display a higher than anticipated rate of increase for CoV and SD of travel time when V/C ratio operated at higher values above 0.8. An unrealistic sharp increase in dependent variable (CoV or SD) of these models could cause an overestimation of travel time reliability cost. Two other models namely the DM and LSCM forecast negative CoV.

A2.1.2 Congestion Index Approach

As the purpose of this study is to look at travel time reliability, a CI approach was utilised instead of the V/C ratio approach (discussed earlier in sub-section 2.2.1, p16). Therefore, Figure A-8 was converted into Figure A-9 below by using Equation A.12. A CI value below 1 is an indication of vehicles operating at higher than the speed limit. The most appropriate models would be expected to show that an increase in CI should not decrease the CoV for travel time.

The findings based on the application of a CI approach produced similar results as the V/C approach in the earlier in Figure A-8.

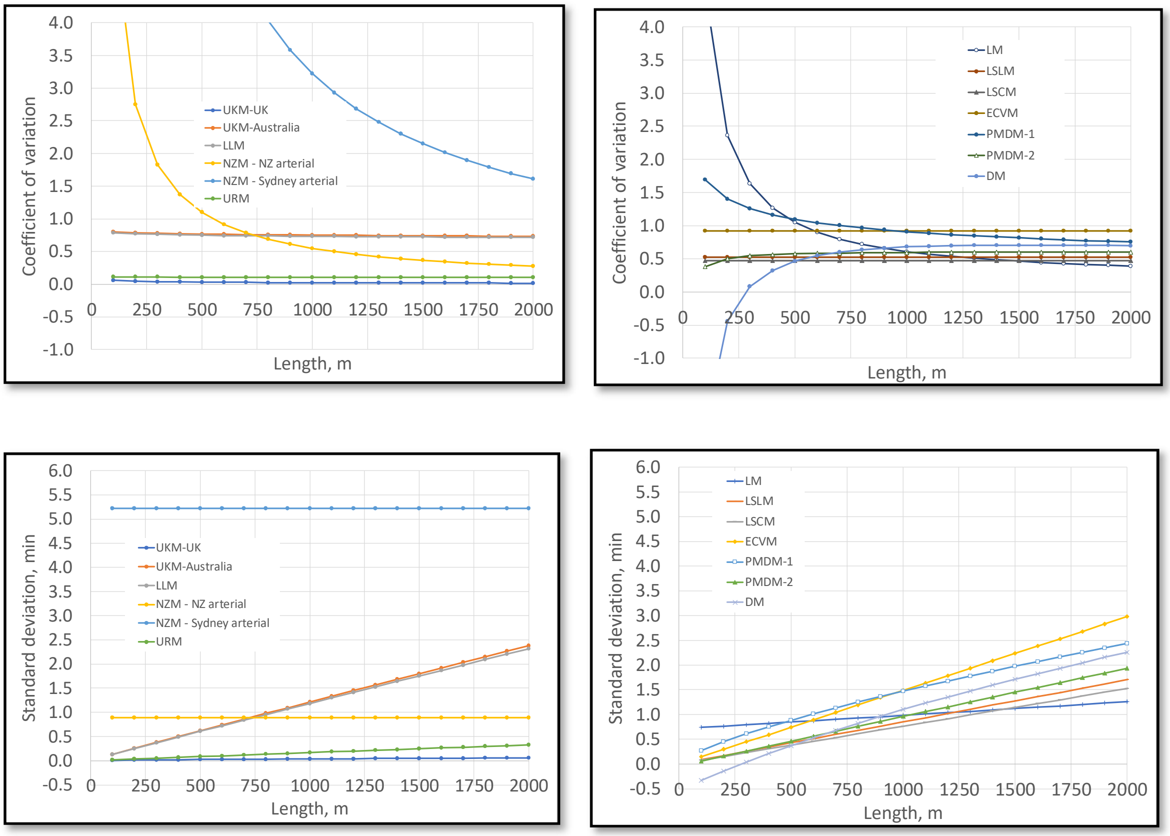
Figure A-9: CoV and SD of travel time versus congestion on arterial highway – existing models



A2.1.3 Link Length Approach

Finally, a numerical analysis on how link length could potentially affect the CoV and SD of travel time for each of the models was undertaken. Figure A-10 presents the outputs of the eleven models. The NZM and LM models tended to return a high value of coefficient of variance when link lengths are short while other models produced more stable values. As demonstrated in Equations A.7 and A.11, the LSCM and DM contain some negative parameters, such as “a” and “d” in DM and ‘a”, “b” and “d” in LSCM. These models were flagged due to having negative outputs in CoV and SD of travel time for link lengths below 280m. All models gave higher SD of travel time for longer link lengths except for the NZM.

Figure A-10: CoV and SD of travel time versus length of travel on arterial highway



A2.2 Comparing Shortlisted Models

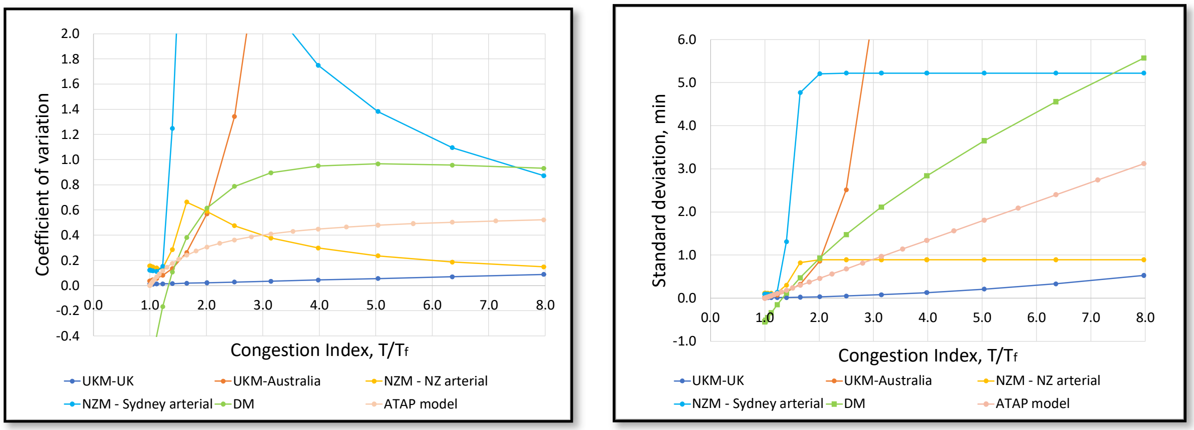
This sub-section of the appendix presents the numerical experiment, discussed in the sub-section 3.1.2 of the report, involving the ATAP link model and the other three shortlisted models (the UKM, NZM and DM). The characteristics of the hypothetical segment (an arterial) considered in the numerical experiment have been discussed in sub-section 2.2.1 of the report. For the UKM, both the UK and Australian versions were modelled. Similarly, for the NZM, both the NZ arterials and Sydney arterials were modelled. In other words, a comparison is made over six models instead of four stated earlier.

As the NZM and DM consider SD while UKM and ATAP model treat CoV as the dependent variable, two sets of plots were developed to compare the results:

- CoV versus CI, and
- SD versus CI.

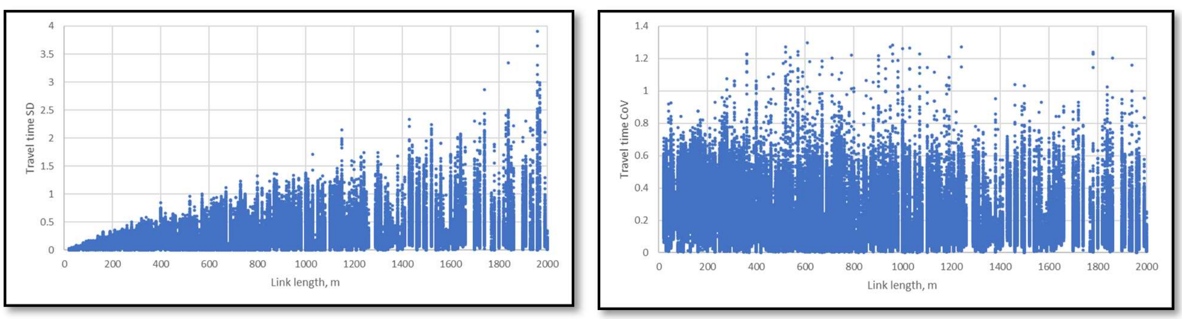
Figure A-11 presents each of the models where the CI is the independent variable (on X-axis). The plot on the left-hand side shows its relationship with respect to the CoV while the one on the right is for the SD. As shown in the figure, UKM-UK forecasts a very low rate of change in the CoV and the SD which is not consistent with the real-world. The UKM-Australia estimates an unbounded and sharply increasing CoV and SD which is unrealistic. The NZM-Sydney also has a vertical and an unbounded CoV but forecasts a constant SD value as CI increases. Similarly, NZM-NZ also shows a stable SD as CI increases and shows a decreasing trend in the CoV. The DM shows a reasonable curve for both the CoV (a stable value at higher CI) and the SD (increasing steadily with CI), however the model predicts negative values at lower congestion. The ATAP link model shows similar curves as the DM and does not give negative values. Furthermore, the slopes of the curves are steadier when compared to the DM which forecasts an abrupt transition. Thus, based on the above discussion, the ATAP link model does reasonably well when compared to other shortlisted models.

Figure A-11: CI vs. CoV or SD of travel time – numerical experiment



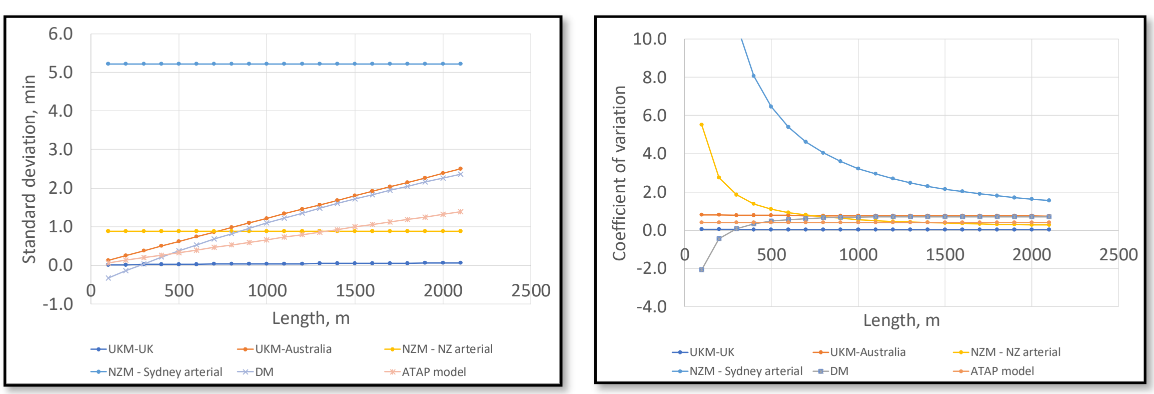
In addition to the numerical experiment, the Perth NetPReS data was utilised to show the observed relationship between travel time SD and CoV with link length in the real-world data, as shown in Figure A-12. Figure A-13 plots the trendlines of the six models to allow for direct comparison against the NetPReS trend. The NerPReS trend in Figure A-12, where the variation in travel SD increases as the link length increases while link length does not appear to impact on the variation in CoV, confirmed our previous findings that travel time SD is length dependent and travel time CoV is not length dependent.

Figure A-12: SD & CoV vs link length < 2km – NetPReS Dataset



The trend in Figure A-12 above is used as a benchmark for best-fit model selection. The best-fit model should present reasonable consistent trend as the Perth NetPReS trend. By comparing different models' SD and CoV trend in Figure A-13, where CoV should be length independent and SD should be length dependent, only the UKM-Australia and ATAP model matched this condition.

Figure A-13: SD & CoV vs link length – numerical experiment



In consideration of the limitations of those shortlisted models identified in Section 3.1.2 and above, the ATAP link model was deemed to be the final selection of best-fit model for travel time SD estimation on a link.

A2.3 Determining Correction Factor in ARSD Approach

The conceptualisation of the ARSD approach, presented earlier in sub-section 3.3.1, was motivated by hypothesising that it may be adequate to approximate route travel time SD as the sum of the SD of the constituent links multiplied by a global correction factor. If this can be shown to be sufficient for determining route choice and link flows, then the incorporation of the travel time reliability in network assignment can be greatly simplified. This could be done by amending the link cost function with an additive term that is proportional to the SD of link travel time (e.g., Equation 3.4). It is only necessary that the ARSD approach be sufficient for route choice modelling to determine links flows. Once links flows are determined the route travel time SD can be calculated using more accurate methods (that is, Equation 2.3 and Equation 2.4).

To examine the ARSD approach, the correction factor (γ) in Equation 3.4 was tested through a numerical experiment using Equation 2.3 and Equation 2.5. Nicholson (2015) determined that the parameter ‘a’ in Equation 2.5 to be equal to -0.05. This represented the level of correlation between the links in a real-world route. The route considered in the numerical experiment was assumed to be composed of multiple links each of length 300m. A variety of scenarios, as summarised below, were then developed and tested:

- Scenarios where individual links are highly (*a* = -0.01), moderately (*a* = -0.05) and weakly (*a* = -0.1) correlated
- Route length of 3 km (that is, 10 links), 9 km (that is, 30 links) and 15 km (that is, 50 links)
- Travel time SD for links were either varied (0.2 s ~ 3.1 s) or constant (1.65 s).

The results from this numerical experiment are presented in Table A-12.

Table A-12: Correction factor (γ) values from the numerical experiment

Link SD	**Level of correlation (‘a’ in Equation 2.5)**	**Correction factor (γ)**		
		3 km route	**9 km route**	**15 km route**
Link travel time SD varies from 0.2 s to 3.1 s.	Highly correlated (*a* = -0.01)	0.99	0.96	0.93
	Moderately correlated (*a* = -0.05)	0.94	0.83	0.75
	Weakly correlated (*a* = -0.1)	0.88	0.72	0.61
Links travel time SD is uniform at 1.65 s.	Highly correlated (*a* = -0.01)	0.98	0.95	0.92
	Moderately correlated (*a* = -0.05)	0.92	0.80	0.71
	Weakly correlated (*a* = -0.1)	0.86	0.68	0.57

The correction factor (γ) in Equation 3.4 is the ratio of route travel time SD over the sum of SD of links on route in the table above. Key observations from the numerical experiment were as follows:

- The correction factor is only marginally sensitive towards the SD of travel time of links. Thus, how the SD varies over a route does not significantly affect accuracy.
- The correction factor is somewhat sensitive to the correlation between the links. A constant correction factor is more accurate in the case where links are more correlated. On the other hand, a constant correction factor would be less accurate in the case where the links are less correlated. If the correlation between the links is consistent across the network, then the impact of the level of correlation on accuracy would be marginal.
- The correction factor is somewhat sensitive towards the distance of travel. A constant correction factor would overestimate the travel time SD on short routes by 15%. On the other hand, travel time SD for longer routes would be underestimated by 12% (that is, assuming a moderately correlated network).

The numerical experiment presented above neither calibrates nor validates the ARSD approach. It can be done using the available route and link travel time SD data for a given jurisdiction. The aim of this experiment was to show the merits to further investigate the application of the ARSD approach, given its ease of implementation in the network modelling exercise. The ARSD approach could be extended to consider different functional forms for the correction factor to improve model accuracy. However, such extensions are beyond the scope of this report.

A3 Datasets Used

This appendix gives a summary of the various datasets that were used to calibrate the link, route and network level models proposed in this report. The datasets used correspond to different regions in Australia which indicate richness of analysis and consistent application of the models across jurisdictions in Australia. The datasets used are: Perth Network Performance Reporting System (NetPReS) dataset from Main Roads Western Australia (MRWA), Gold Coast National Performance Indicators (NPI) and Bluetooth (BT) data from the Department of Transport and Main Roads (TMR) Queensland, and Sydney travel time data from Google. The datasets are discussed below:

A3.1 Perth NetPReS Data

The Perth NetPReS data originally consists of 29 arterial and freeway routes in both directions. Figure A-14 shows the map of routes for which data was available along with Table A-13 which provides the route names. The range of link lengths that make up the Perth network, span between 20 m and 37,920 m. 22 links, which are greater than 10 km in length, were not considered typical metropolitan links and were excluded from the analysis. As a result, two arterial routes were excluded, and the remaining 27 routes were used for the analysis.

Figure A-14: NetPReS dataset metropolitan routes for Perth network

The characteristics of filtered datasets are summarised below:

- Covers metropolitan Perth area
- Comprises bi-directional speed and volume data for arterials, controlled-access highways and freeways
- Speed data was collected from multiple sources such as TomTom, AddInsight, NPI, IRIS, Intelematics
- Data duration: 4 months (from 1 August 2018 to 31 November 2018)
- Data resolution: every 15 min between 5am and 9pm.
- Number of links: 947

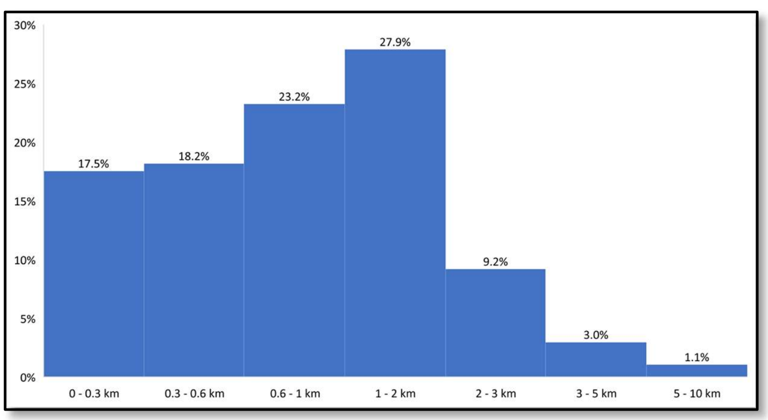
- Total road length: 1076 km.

Table A-13: Route information in NetPReS data

Route ID	Route Name	Route ID	Route Name
1	Albany Hwy	16	Mitchell Fwy
2	Armadale Rd	17	Orrong Rd
3	Brookton Hwy	18	Reid Hwy
4	Canning Hwy	19	Roe Hwy
5	Cockburn Rd	20	South St
6	Graham Farmer Fwy	21	South Western Hwy
7	Great Eastern Hwy Inner	22	Stirling Hwy
8	Great Eastern Hwy Outer	23	Thomas Rd
9	Great Northern Hwy	24	Tonkin Hwy North
10	Guildford Rd	25	Tonkin Hwy South
11	Karrinyup-Morley Hwy	26	Toodyay Rd
12	Kwinana Fwy	27	Wanneroo Rd / Indian Ocean Dr
13	Leach Hwy	28	West Coast Hwy
14	Marmion Av	29	Port Access
15	Melville Mandurah Hwy		

Figure A-15 shows the distribution of links by link lengths in the filtered dataset. The figure shows that 58.9% of the links in the dataset are short links of less than 1 km in length, and 1.1% of the links are long links greater than 5 km in length.

Figure A-15: Distribution of links in Perth NetPReS Dataset by link length



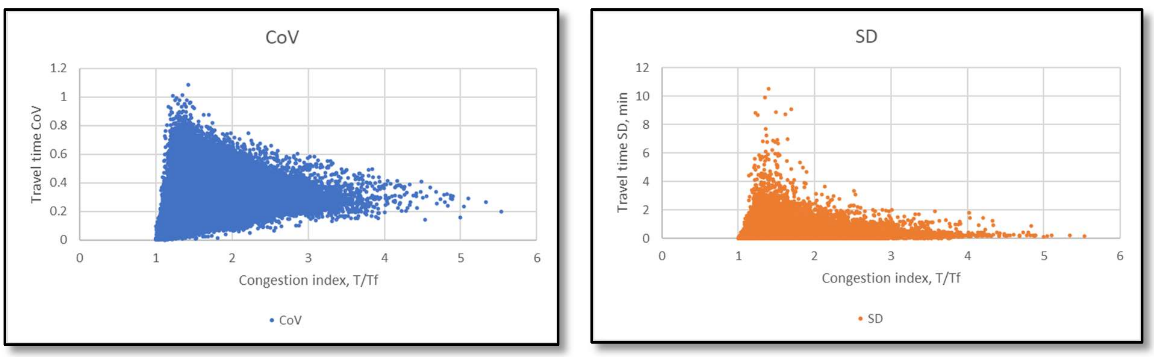
Examination of the raw speed data was conducted to identify potential outliers, as a slow speed observation could significantly impact the travel time SD value and potentially skew the results. Inspection of the cause of the slow speed observations did not reveal any unusual event such as extreme weather or incidents. It was considered appropriate to consider the speeds below 10 km/h as unsuitable and excluded from the analysis. 15,803 out of 5.66 mil (0.28%) observations were excluded as a result.

It was identified in the literature review that extreme weather conditions and major incidents that involved multiple lane closures have a significant impact on traffic volume and speed, particularly travel time reliability. As this study is focused on facilitating a robust comparison between month to month travel time reliability, days that experienced major incidents or weather events should be excluded from the analysis. A search for major incidents and extreme weather events was conducted, and no event was showed over the data period.

A series of plots were then developed comparing CI against each of CoV and SD of link travel time to facilitate empirical analysis of the available data. It was found that a small percentage (0.2%) of observations showed CoV and SD value of 0 due to the reason that the speed was patched with identical values. It was considered appropriate to exclude them from the analysis. Figure A-16 shows the plot for all available links that are less than 10 km in length. Similarly, a comparison was conducted for the following indicators:

1. Direction of travel
2. Link length
3. Time of day
4. Road type.

Figure A-16: Travel time CoV against SD for links in NetPReS Dataset - arterial



The CoV figure above shows a 'triangle' shape. The CoV starts from 0 when CI equals 1. The value of CoV gradually increases as the congestion index increases. However the figure shows that the range of CoV become wider as the CI increases, and when CI reaches a certain point, e.g. CI=1.3 on the graph, the range of CoV becomes narrower as the CI continues to increase. The centre point of CoV range for CI value greater than 1.3 is constant at approximately 0.4 or slightly below 0.4. This finding is consistent with the literature that "*at very low traffic volumes the traffic state could be 'unstable' and the variability of travel times might be higher than expected. As traffic volumes increases, the traffic state is becoming 'stable' up to the local minimum point. From this point the traffic starts to become heterogeneous and travel time variability is increasing with the saturation level. Then towards a heavy congestion state, traffic is about to become homogenous due to queuing in which state the variability of travel times is decreasing.'* PIARC (2019). This can also be used to explain the observation in the SD figure. Furthermore, there is a limited representation of points with CI value greater than 4 which indicates that instances of excessively long travel times (denoted by CI greater than 4) are infrequent which makes sense due to the prevailing traffic characteristics in the Perth metropolitan area.

Figure A-17 shows the comparison based on the direction of travel. The trend and magnitude for CoV and SD of travel time are shown to be similar for both directions. Thus, at a macro-level (e.g. network), it is expected that a generalised set of parameters can be applicable to both inbound and outbound data.

Figure A-17: CoV against SD for two directions of travel in NetPReS Dataset – arterial

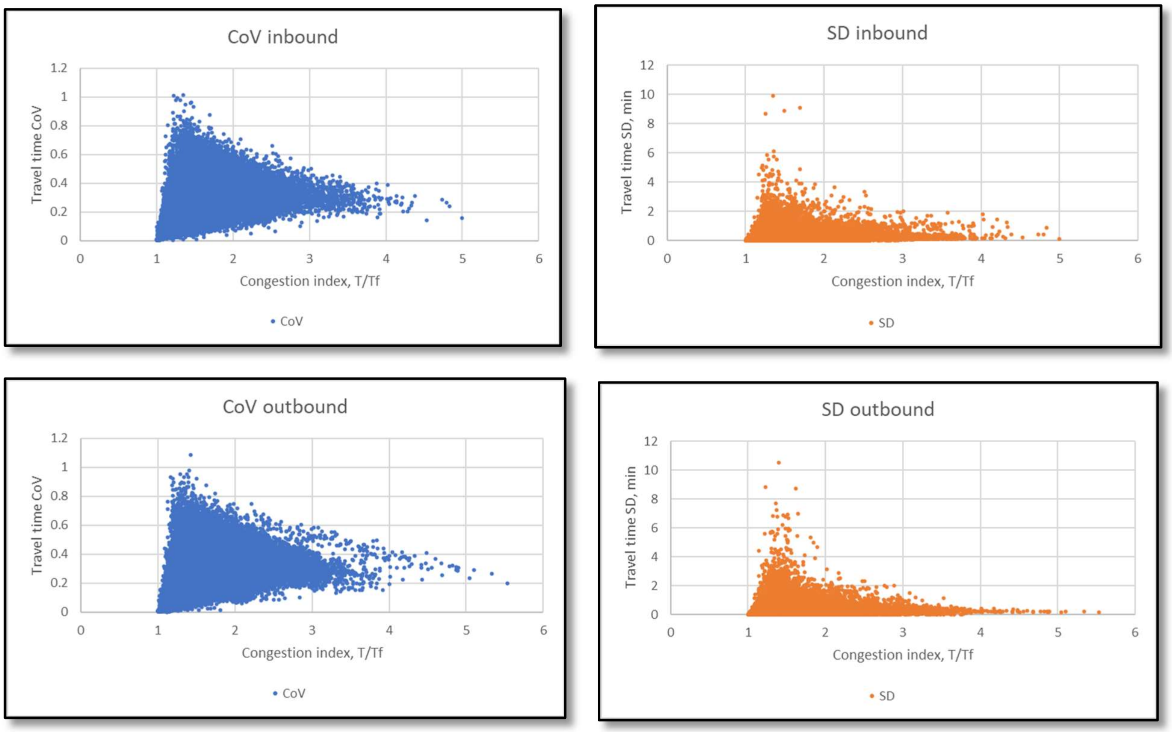


Figure A-18 compares the CoV and SD for different categories of link lengths. The figure shows that while the link length does not have an impact on the slope of the CoV curves or the shape of the CoV graph, it however does have an impact on the SD curves. The slope of the SD curve increases as the link length is increased, which again is consistent with previous findings in the literature. The longer a link gets, the more it is prone to experience delays (due to more incidents, traffic lights, etc.) thus adding to travel time variability. The slope of the SD curve increases as the link length is increases. The CoV, on the other hand, appears to be length independent. It can also be mathematically proven why CoV is length independent:

$$\boldsymbol{CoV = \frac{\sigma}{T} = \frac{SD}{Mean\ travel\ time}}$$

$$\boldsymbol{T = \frac{L}{V} = \frac{Length}{Speed}}$$

$$\boldsymbol{CoV = \frac{\sigma \times V}{L}}$$

The normalisation of travel time SD, which depends on link length, divided by link length makes CoV independent of the link length. Thus, at a network level, a generalised set of parameters for CoV estimation should be sufficient for all link lengths.

Figure A-18: CoV against SD for different link lengths in NetPReS Dataset – arterial

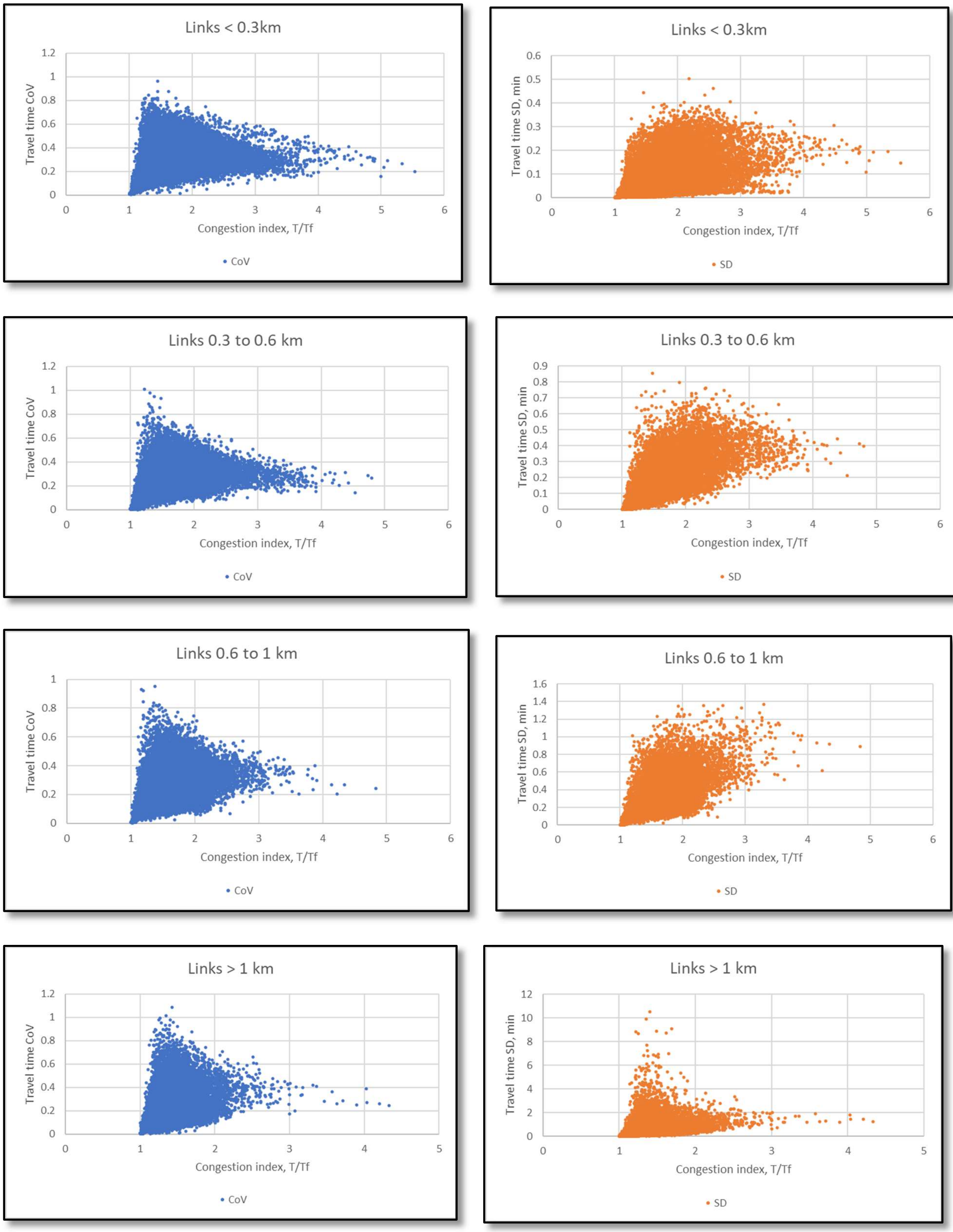
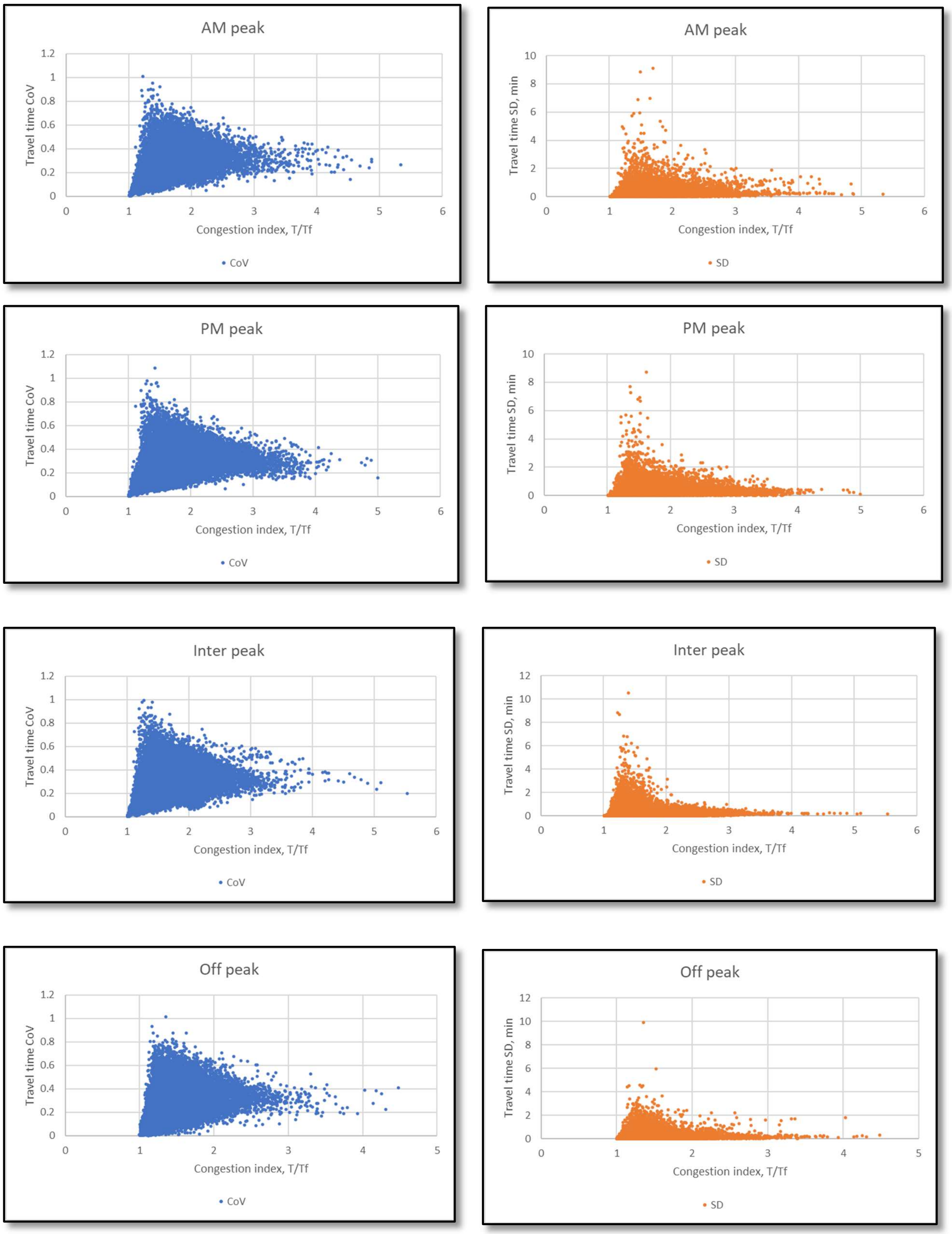


Figure A-19 shows the split of observed data by the time of the day. The data points for the different time-periods does not appear to have an impact in both CoV and SD graphs, which indicates that a generalised set of parameters can be applicable to model different time-periods.

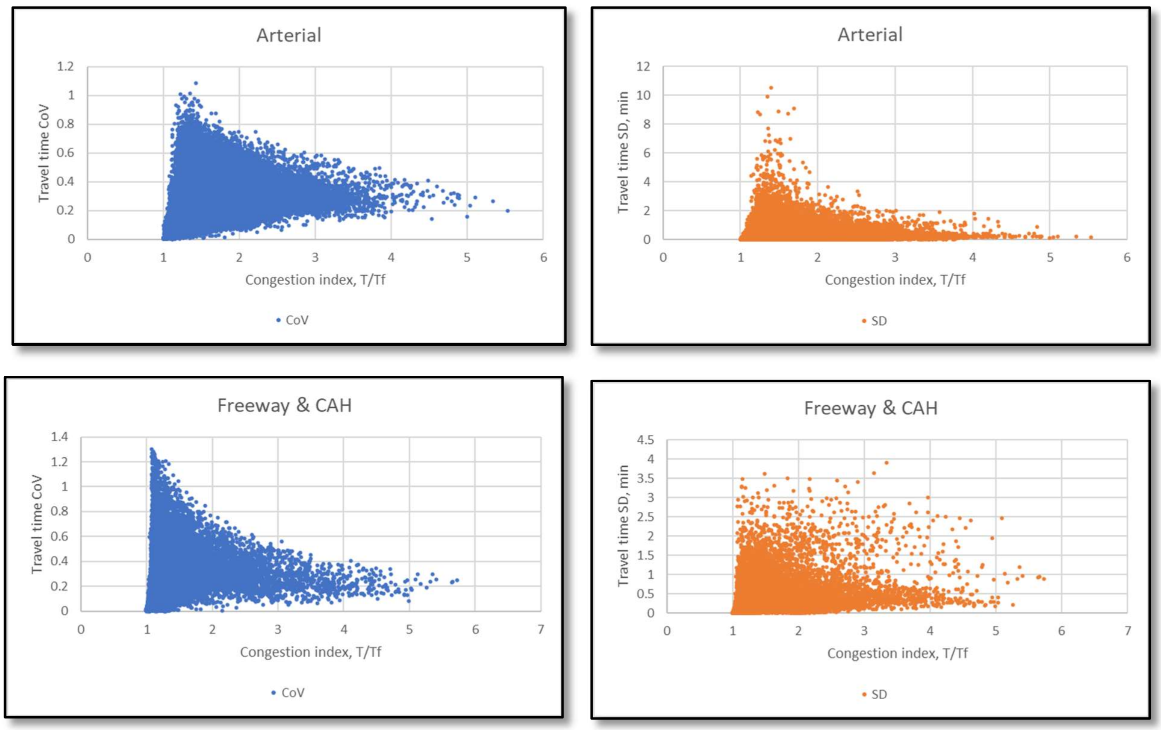
Figure A-19: CoV (LHS) and SD (RHS) against CI for different time-periods in NetPReS Dataset – arterial



Similar analysis was also conducted on freeway data, with the findings being consistent with arterial data. The travel direction, link length and time-period do not appear to have an impact on CoV values, which again indicates that a generalised set of parameters can be applicable to model different travel directions, link lengths and time-periods.

Figure A-20 shows the comparison based on road type. The Perth road network is made up by three road types: arterials, controlled access highways (CAH) and freeways. The CAH and freeways were combined since they depict similar characteristics and were then compared against the arterials. The magnitudes of CoV and SD of travel time are quite different between the road types, thus justifying the need of two sets of parameters for different road types. The CoV curve for freeways is slightly steeper than that of arterials, the CoV value for freeway peak is earlier than arterial, e.g., at CI=1.1 for freeway vs at CI=1.3 for arterial, and the spread of CoV values narrow at faster rate than arterial as the CI value continue to increase. However, the number of datapoints for freeways and CAH are less when compared to that of the arterials.

Figure A-20: CoV and SD against CI for different road types in NetPReS Dataset



A3.2 Gold Coast NPI and Bluetooth Data

Figure A-21 shows the study site locations and boundaries of the Gold Coast NPI data. This network is made up of a series of local roads, arterial and freeway routes in both directions. The specifications of the dataset are summarised below. Data was converted into 15 min resolution to allow for direct comparison between Perth and Gold Coast data. The data conversion method applied was the sum of three 5-min volumes as new volume and volume weighted average speed. The characteristics of the datasets are summarised below:

- Covers metropolitan Gold Coast area
- Comprises NPI speed and volume data, Bluetooth speed data
- Arterials and freeways administered by the TMR, arterials and local streets by the councils
- Data duration: 1 month (February 2019)
- Data resolution: 5 min collected 24/7.

Figure A-21: Available NPI data available for Gold Coast network

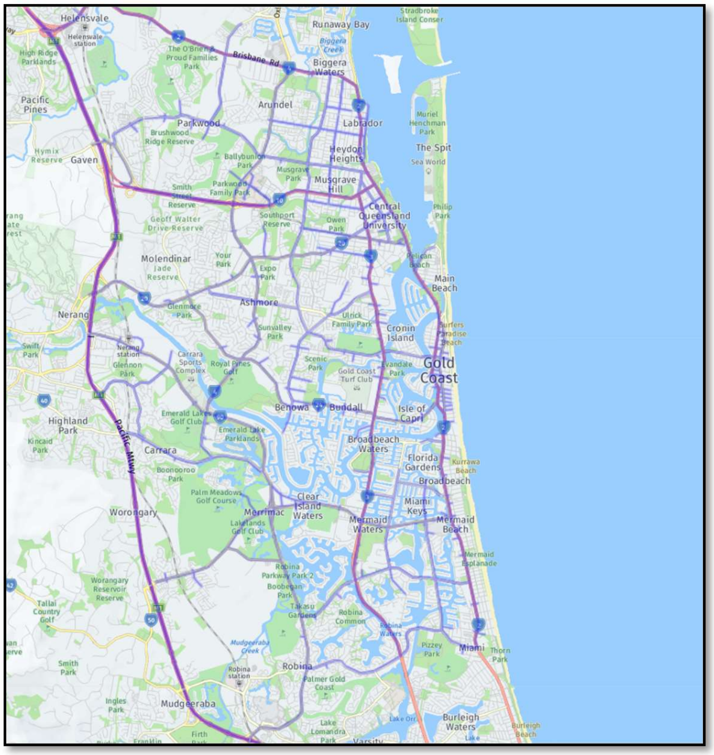
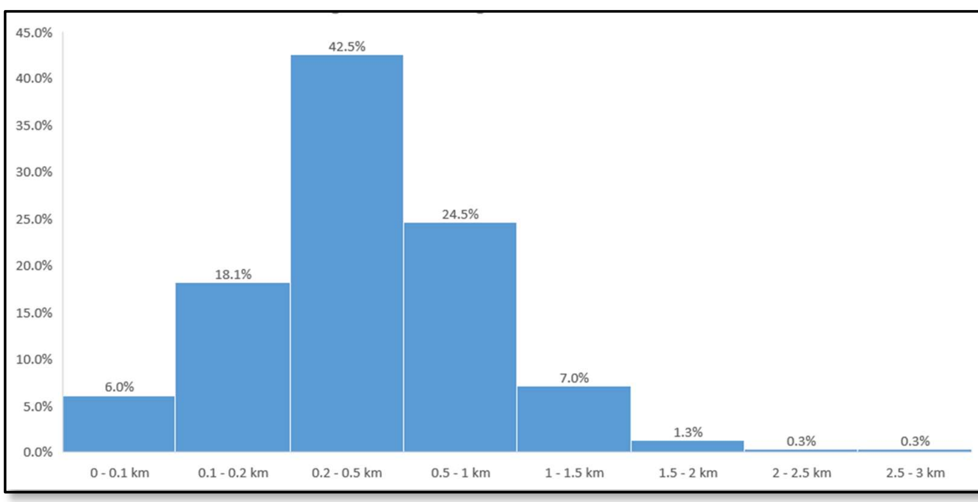


Figure A-22 shows the distribution of links based on link lengths in the Gold Coast network. All the links are up to 3 km long. When compared to the Perth network, the Gold Coast data has a greater proportion of shorter links (less than 0.5 km in length). In terms of the total percentage, 42.5% of Gold Coast links are less than 0.5km in length compared to 30.1% for Perth. The reasons for this variation could be geographical differences and the way links are defined and scoped across the two jurisdictions.

Figure A-22: Gold Coast network link length histogram



The main difference between the Perth and Gold Coast datasets is the road types in the dataset. The Perth dataset only contains State controlled arterial and freeways. The Gold Coast dataset contains both state controlled arterials and freeways and Council controlled arterials and side streets. Most of the links in Gold Coast comprise arterials and the side street data are less than 0.5 km in length.

An initial assessment of the side streets data failed the data quality test. Side streets had much lower priority and tended to give way to main road, therefore the slower speed on side street did not indicate a congestion and congestion index in this case could cause a false impression of congestion. A discussion with TMR representatives concluded that this side street data should be excluded from analysis. ARRB (2018) in R47 NACoE project using similar dataset also concluded that the short NPI link length configured for side-streets tended to present a problematic profile when investigating travel time variability.

Table A-14 outlines the number of links per road type for the Gold Coast dataset as well as the data quality assessment results whether to be used for the analysis.

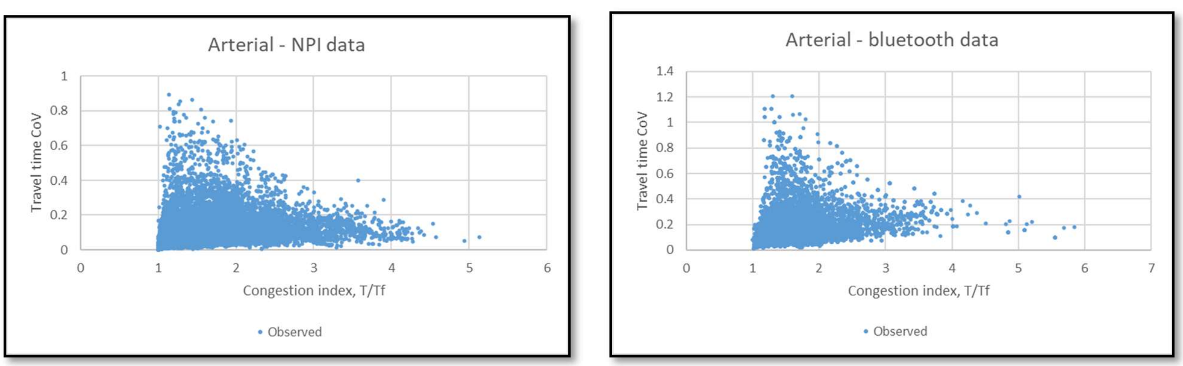
Table A-14: Gold Coast dataset road type

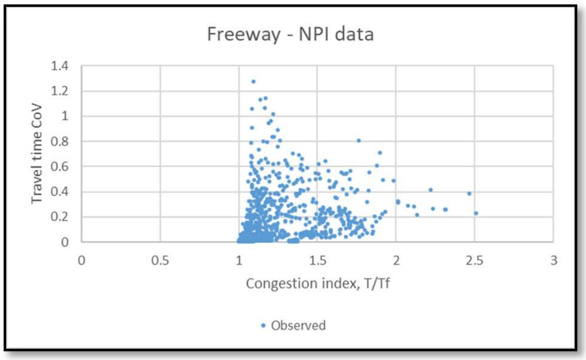
Gold Coast Road type	Number of links	As % of total link number	Used for analysis (Y/N)
Motorway (TMR)	46	5.9%	Y
Arterial (TMR)	231	29.5%	Y
Arterial (Council)	168	21.5%	Y
Side Street (Council)	338	43.2%	N

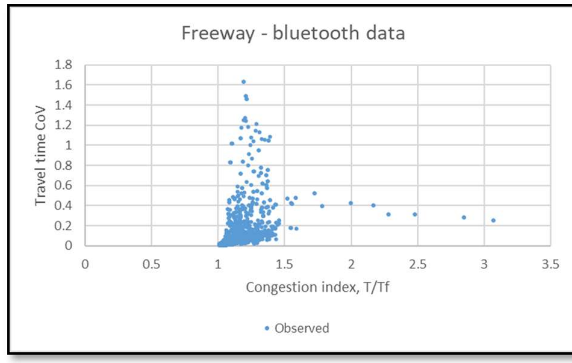
The same filters that were used in Perth data were also applied to the Gold Coast data, namely exclude CoV values that equalled 0 and speed observation below 10 km/h. Figure A-23 shows the comparison based on road type side-by-side using two different data sources. For the NPI data on the left, even though the shape of the CoV distribution was still a 'triangle' as found in the Perth data, the lower bound CoV value for high CI was unexpectedly low, at CI equal 3.35, the CoV value equal 0.02. As the CI increases, the value of CoV tends to reduce towards 0. The possible explanation for this could be due to the nature of NPI arterial data been modelled rather measured. The modelled speed might not necessarily represent the true speed and introduces greater uncertainty. Therefore, to avoid potential misleading results, Gold Coast Bluetooth data was also utilised to allow direct comparison with the NPI data.

The freeways show a visibly different trend when compared to arterials. There is a lack of datapoints for freeways beyond a CI of 2 which may affect the ATAP link model goodness of fit for the dataset. The difference in the shape of CoV distribution between arterial and freeway indicated that two sets of parameters are required for different road type, which is consistent with the Perth dataset.

Figure A-23: CoV Against CI for different road types in NPI and bluetooth data

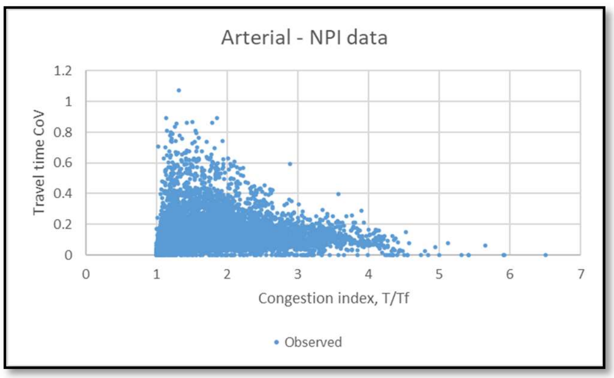






To elaborate the limitation of NPI arterial data further, Figure A-24 shows the CoV against CI from arterial data without the 0 CoV filter in an attempt to observe the complete trend from NPI arterial dataset. A clear trend is now seen as the CI value increases, the tendency of CoV reduce to zero is very high. This is deemed to be the limitation of NPI data which cannot be observed from field data.

Figure A-24: CoV against CI from NPI arterial data – CoV of 0 included



Through the comparison of Gold Coast NPI speed data with Bluetooth speed data in this section, Bluetooth data was deemed to provide better speed data quality for the following reasons:

- NPI speed data are modelled rather measured from field
- NPI speed data showed a decrease in CoV (to zero) as CI increased (see Figure A-24), which contrasted with the trends observed in other datasets.

Therefore, Bluetooth data was more suitable for the purpose of this project and was utilised for the validation of ATAP model.

A3.3 Sydney Google Data

Moylan et al. (2018) collected a variety of data for the Sydney Greater Metropolitan Area (GMA) to study travel time reliability. This project utilised the travel time data from Moylan et al. (2018) which was collected using the Google Maps Directions API. Travel times prevailing on 37 routes, 74 routes when considering bi-directional movement, spread across the study area were collected. The data was collected by pinging the Google Maps Directions API 55 times a day, storing the real-time travel time value provided under the field 'duration in traffic' in the API, for a period of 14 months (February 2017 – March 2018). Figure A-25 shows the spatial locations of the 37 routes selected for analysis by Moylan et al. (2018). Table A-15 lists the names of these routes which includes a few major arterials and motorways in Sydney.

Figure A-25: Routes chosen in the Sydney GMA (source: Moylan et al. (2018))

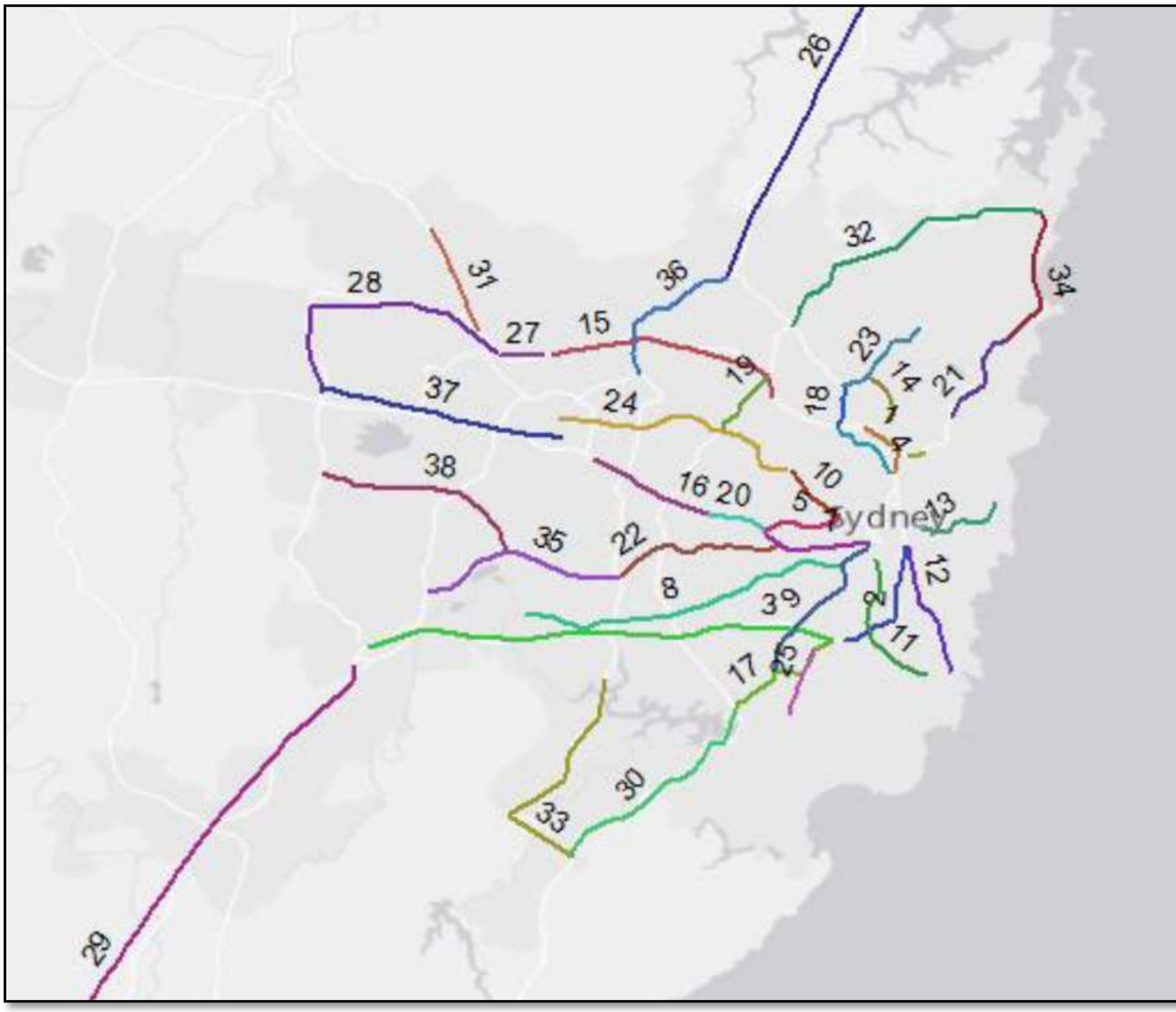


Table A-15: Description of the chosen routes

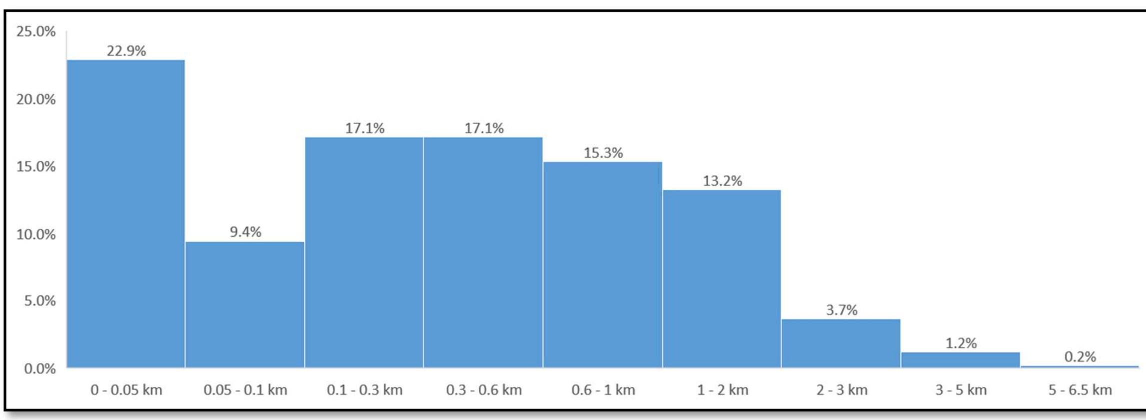
Road ID	Name	Start Point	End Point	Route ID	Location
M1	Gore Hill Freeway	Lane Cove	North Sydney	1	Inner
M1	General Holmes Dr	Surry Hills	Mascot	2	Inner
M5	M5 & M5 East	Casula	Mascot	3	Inner
A1	Pacific Highway	Lane Cove	North Sydney	4	Inner
A4	City West Link	Rozelle	Haberfield	5	Inner
A8	Military Road	Balgowlah	Cammeray	6	Inner
A22	Parramatta Road	Ashfield	Sydney	7	Inner
A34	Milperra Road	Newtown	Liverpool	8	Inner
A36	Princes Highway	Arncliffe	Haymarket	9	Inner
A40	Victoria Road	Rozelle	Hunters Hill	10	Inner
	Botany Road	Haymarket	Marrickville	11	Inner
	Anzac Parade	Waterloo	La Perouse	12	Inner
	New South Head Road	Potts Point	Vaucluse	13	Inner
	Eastern Valley Way	Roseville Chase	Cammeray	14	Inner
M2	Hills Motorway	Seven Hills	North Ryde	15/27	Middle/Outer
M4	Clyde	Concord	16	Middle	
A1	Princes Highway	Blakehurst	Arncliffe	17	Middle
A1	Pacific Highway	Roseville	Lane Cove	18	Middle

Road ID	Name	Start Point	End Point	Route ID	Location
A3	Lane Cove Road	North Ryde	Ryde	19	Middle
A4	Parramatta Road	Concord	Ashfield	20	Middle
A8	Pittwater Road	Brookvale	Balgowlah	21	Middle
A22	Liverpool Road	Bankstown	Ashfield	22	Middle
A38	Warringah Road	Roseville	Frenchs Forest	23	Middle
A40	Victoria Road	Parramatta	Hunters Hill	24	Middle
	The Grand Parade	San Souci	Mascot	25	Middle
M1	Pacific Motorway	Morisset	Wahroonga	26	Outer
M7	Westlink Motorway	Eastern Creek	Seven Hills	28	Outer
M31	Hume Highway	Wilton	Casula	29	Outer
A1	Princes Highway	Heathcote	Blakehurst	30	Outer
A2	Old Windsor Road	Rouse Hill	Bella Vista	31	Outer
A3	Mona Vale Road	Mona Vale	Pymble	32	Outer
A6	New Illawarra Road, Alfords Point Road	Padstow	Heathcote	33	Outer
A8	Pittwater Road	Mona Vale	Brookvale	34	Outer
A22	Liverpool Road	Liverpool	Bankstown	35	Outer
A28	Cumberland Highway	Wahroonga	Carlingford	36	Outer
A44	Great Western Highway	Eastern Creek	Parramatta	37	Outer
	The Horsley Drive	Horsley Park	Lansvale	38	Outer

Source: Moylan et al. (2018)

The Sydney network is made up by 1256 links, with link length spans between 1 m and 6,279 m. Figure A-26 shows the distribution of links based on link lengths in the Sydney network.

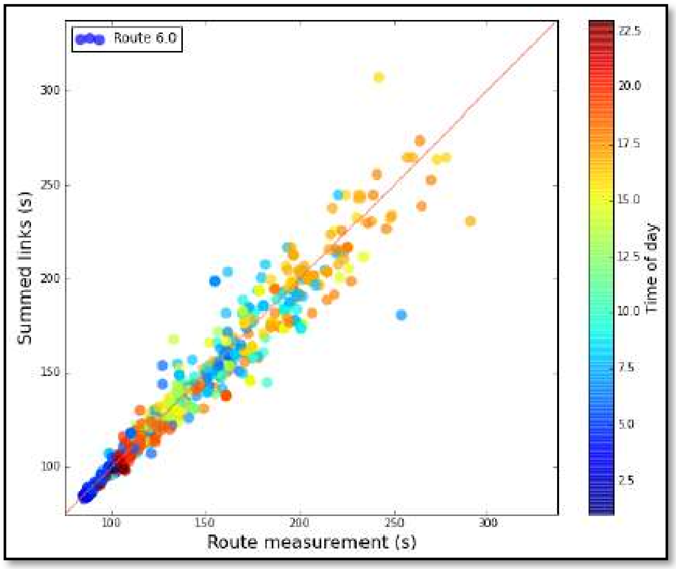
Figure A-26: Sydney network link length histogram



The travel times on the links forming these routes was also collected using the Google Maps Distance Matrix API which provided the shortest-path travel time between the starting and the ending point on a link. In addition to Google data, traffic counts data, including motorway loop detectors were also collected as part of this study.

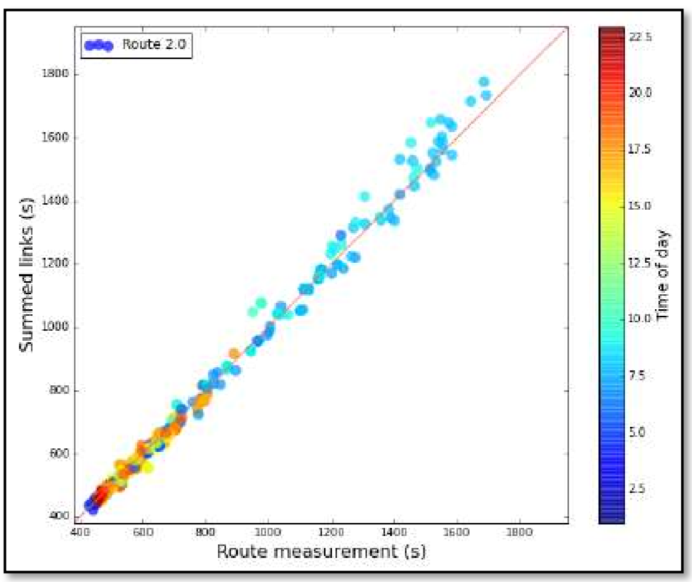
For each query time for each route in the Sydney case study, the component links were summed up to estimate an instantaneous travel time. The summed-link and route travel times were compared in Figures A-27, A-28 and A-29. Even at the extremes of observed conditions, the correlation between the two measures were strong. Arterial routes (e.g., Route 6.0, Military Road) tended to show more scatter than motorways (e.g.. Route 2.0, M1 Motorway). Most routes in the study do not show any systematic biases between the route travel time and the summed link travel times, but the most common disagreement is that the summed links tend to overestimate the travel times during periods of congestion (for example, Route 11, Botany Road).

Figure A-27: Comparison of route and summed link travel times for an arterial route (Military road)



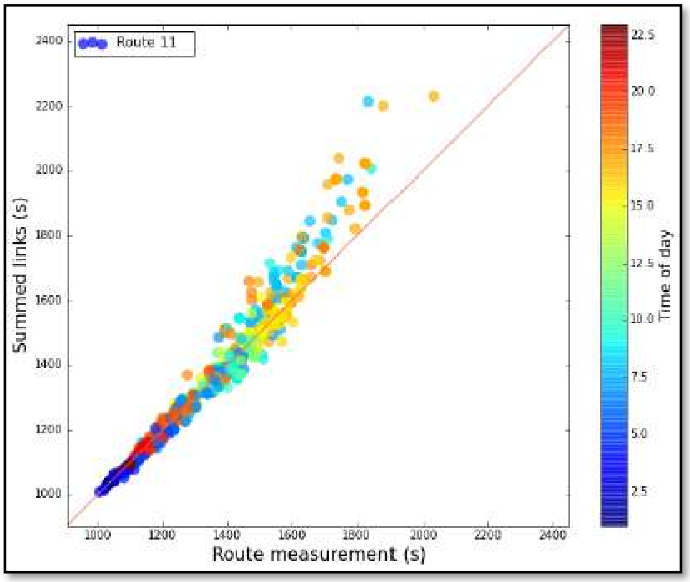
Source: Moylan et al. (2018)

Figure A-28: Comparison of route and summed link travel times for a motorway (M1)



Source: Moylan et al. (2018)

Figure A-29: Comparison of route and summed link travel times for an arterial route (Botany road)



Source: Moylan et al. (2018)

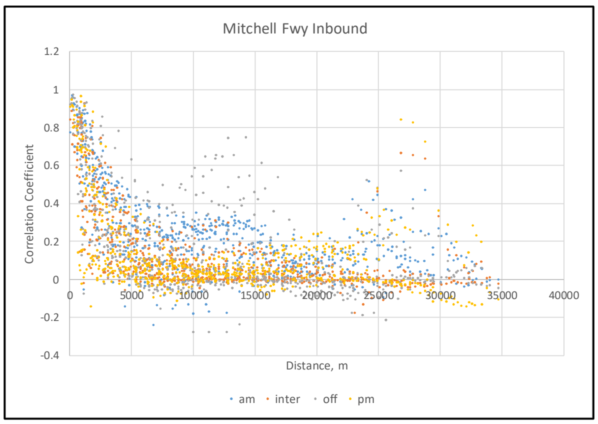
A4 Route Correlation Analysis

The aim of the correlation analysis was to verify whether the link travel times are correlated with one another. Once this was verified, the next step was to find the best model form for predicting the travel time correlation coefficient between two given links. This was achieved through first developing the Nicholson correlation coefficient on the available data and comment on the model goodness-of-fit. The preliminary analysis was conducted using the Perth NetPReS data. An alternative model form would be explored if the CCM recommended by Nicholson was not found to be the best model form.

A4.1 Correlation on Freeways or Controlled Access Highways (CAHs)

Assuming link travel times to be independent of one another can significantly underestimate travel time reliability cost (Nicholson, 2015, Moylan et al., 2018). The travel time correlation coefficient between links, based on midpoint to midpoint distance is shown in Figure A-31 for the Mitchell Freeway in Perth (for inbound and outbound data across different peak traffic).

Figure A-31: Travel time correlation coefficient between links – Mitchell freeway, Perth



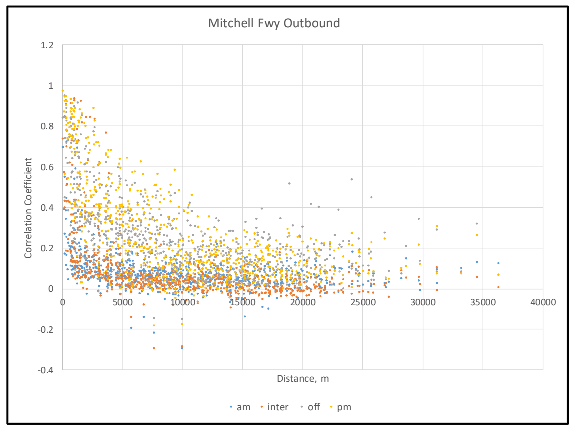
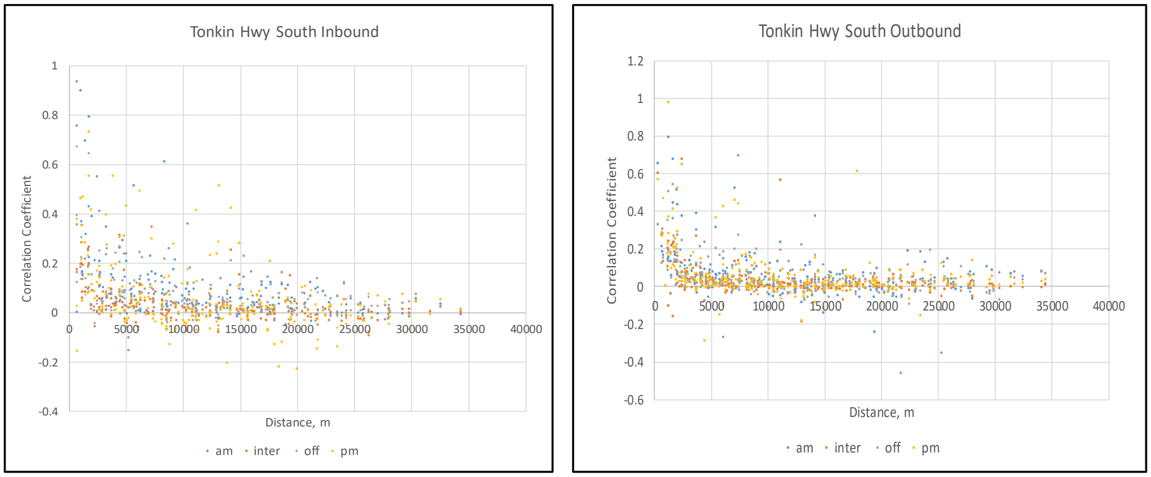


Figure A-31 indicates that that the link travel times are not independent. As expected, links close to each other tend to have a higher travel time correlation coefficient when compared to the links further apart. For the inbound direction, the AM peak has the highest correlation coefficient value between the links, which makes sense, while the PM peak has the lowest correlation coefficient value. Similarly, for the outbound direction, the PM peak has the highest correlation coefficient value between the links which again makes sense as it is associated with people leaving work.

Figure A-32 shows the travel time correlation coefficient plot for a CAH, the Tonkin Highway South. Although it is not as obvious as Figure A-31, the correlation between travel times for different links within a route can still be observed. The time-of-day trend does not appear to be significantly different for different directions. Further analysis is required to confirm whether this is individual case or a case in general. If all the CAHs were to behave similarly, it is recommended to exclude CAHs from the freeway parameters or assign an additional category for CAH to reduce the chance of modelling error and improve model accuracy.

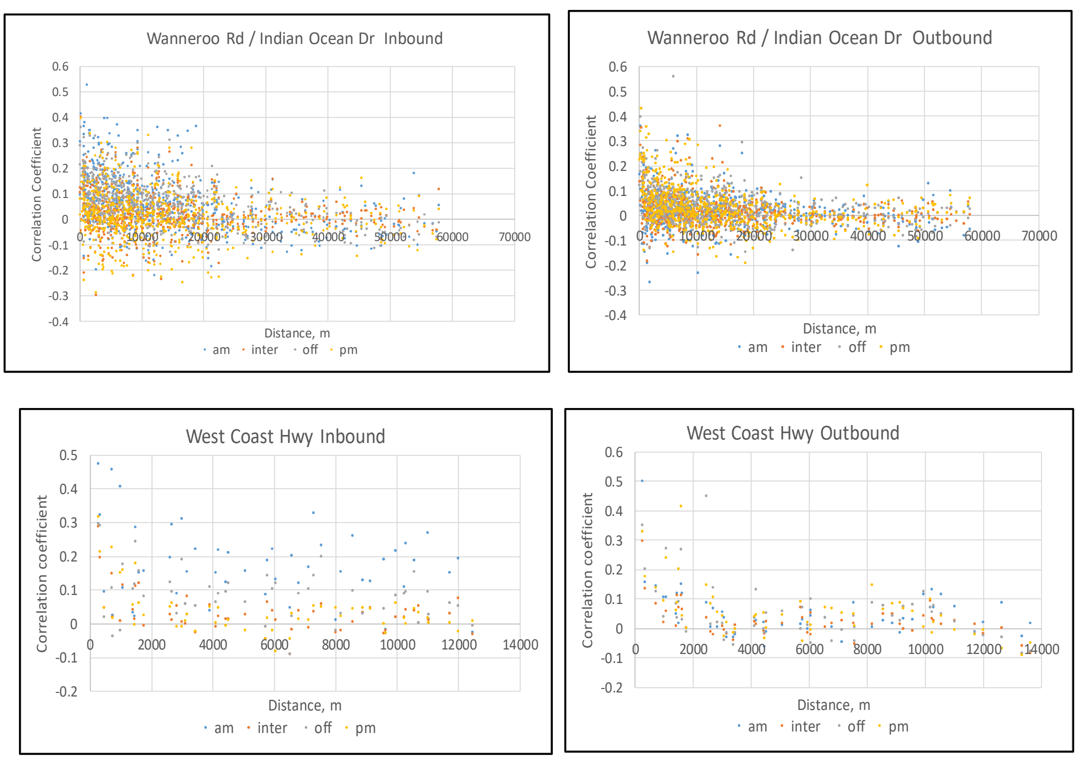
Figure A-32: Travel time correlation coefficient between links – Tonkin highway South



A4.2 Correlation on Arterial

It was initially assumed that an arterial route would have a lesser travel time correlation when compared to a freeway due to a higher exposure to other road users (such as pedestrians and cyclists, school zones, trams, parking, etc) and resulting in a greater variability. Geometrical differences in freeway and arterial road can also contribute to the reduction in travel time correlation. Figure A-33 shows the travel time correlation coefficient between links for two selected arterial routes in both inbound and outbound directions.

Figure A-33: Travel time correlation coefficient between links – arterial



The correlation coefficient value between links for both the inbound and outbound direction on arterial roads does not appear to be significantly different to each other. The time of day for the outbound direction appears to overlap with each other. However, for the inbound direction, both routes show different trends for different times of the day, with the AM peak showing the highest correlation while the PM peak shows the lowest correlation. The magnitude of the correlation (around 0.4) does appear to be significantly lower than freeway (closer to 1) which makes sense due to the reason discussed above. Thus, it can be concluded that the link travel times are not independent and that there is a correlation between the travel times for different links within a route. Freeways were found to have much higher correlation between link travel time compared to arterial roads.

A4.3 Model Comparison

This sub-section presents the calibration results for the other two models, exponential (proposed by Nicholson (2015)) and shifted exponential developed alongside the linear-log CCM (Equation 3.4). Tables A-16 and A-17 present the summaries for the exponential and shifted exponential models respectively. The R-squared values of these two models are lower, in general, than the linear-log CCM presented in earlier Table A-15 (barring some R-squared values for the freeway inbound for the exponential model). This indicates a superior model fit of the linear-log CCM.

A few shifted exponential models (shown in Table A-17) failed to converge within 10,000 iterations of the optimisation procedure, and hence were unable to reach the optimal parameters which minimise the RMSE.

Table A-16: Calibrated parameters for the exponential Model

Road type	Direction	Time-period	a	R^2
Arterial	Inbound	AM peak	-2.51***	-0.2680
		Inter peak	-7.0626***	0.0071
		PM peak	-5.420***	-0.0207
		Off peak	-2.905***	-0.4278
	Outbound	AM peak	-5.2721***	-0.2357
		Inter peak	-7.1432***	0.0116
		PM peak	-3.6246***	-0.0403
		Off peak	-3.1981***	-0.2760
Freeway	Inbound	AM peak	-0.6012***	0.3997
		Inter peak	-0.8310***	0.3954
		PM peak	-0.6975***	0.3453
		Off peak	-0.6941***	0.3445
	Outbound	AM peak	-1.2369***	0.0127
		Inter peak	-1.018***	0.2853
		PM peak	-0.4371***	0.2670
		Off peak	-0.5694***	0.0692

*Note: asterisks denote statistical significance: * at 10%, ** at 5%, *** at 1%.*

Table A-17: Calibrated parameters for the shifted exponential model

Road	Direction	Time-period	a	b	c	R^2
Arterial	Inbound	AM peak	-1130.263	0	1130.385	0.1106
		Inter peak	-373.436	0	373.473	0.0318
		PM peak	-457.1049	0	457.157	0.0511
		Off peak	-867.429	0	867.549	0.1332
	Outbound	AM peak	Model failed to reach convergence			
		Inter peak	-414.276	0	414.312	0.0452
		PM peak	-861.366	0	861.437	0.0937
		Off peak	Model failed to reach convergence			
Freeway	Inbound	AM peak	Model failed to reach convergence			
		Inter peak	-1396.630	0	1396.769	0.1173
		PM peak	-1332.581	0	1332.746	0.1298
		Off peak	Model failed to reach convergence			
	Outbound	AM peak	Model failed to reach convergence			
		Inter peak	-1437.469	0	1437.593	0.1011
		PM peak	Model failed to reach convergence			
		Off peak	Model failed to reach convergence			

*Note: asterisks denote statistical significance: * at 10%, ** at 5%, *** at 1%.*

A5 Application of ARSD

This appendix presents the application of the ARSD method using the Sydney road network (comprising 74 routes (37x2) only) and the Google travel time data given in Moylan et al. (2018). The appendix first describes the outputs from the traditional traffic assignment algorithm, that is UE, on the given information. These outputs are in turn used as inputs to run the ARSD approach. The results from the ARSD method, that is route travel time SD are presented towards the end of this appendix.

A5.1 Inputs to ARSD Method

The standard UE model on the Sydney network resulted in link flows and travel times under equilibrium conditions, that is when no traveller can further reduce the travel time, between an OD pair, by unilaterally shifting between the routes. In other words, the travel time on all used paths between an OD pair are minimum and equal at UE. Figure A-34 shows a snapshot of the output file generated from UE. The first and the second column denote the starting node and the ending node respectively which define a link. Volume is reported in veh/h while travel time is in minutes.

Figure A-34: UE link flows and travel time for Sydney road network

From	To	Volume	TT
1838825566	1832386626	1402.596482	0.02975843
1837573347	1832386626	1571.158738	1.068347524
1832386628	60901426	2931.983274	0.204480524
1833251883	1832386628	2931.983274	0.587469607
60901425	1832386628	2155.652774	0.071381467
6337070259	1811939442	2465.347411	1.127893671
1811939468	1811939442	1093.839882	0.035959198
1817317907	1811939442	46.88782988	0.078948028
1832386680	4041121208	549.0245966	0.030491495
1838825566	1832386680	477.8725455	0.416381848

Apart from the UE outputs, the following network information was also available. Firstly, the existing link level details which are shown in Figure A-35. In this figure, from and to are the start and end nodes defining a link, capacity is the maximum flow on a link (in veh/h), length is the length of the link (in m) and free flow time (in minutes) is the ratio of link length and speed limit (in km/h). Secondly, the mapping information which relates individual links into the 37 routes of interest for which Google travel time data is available. Figure A-36 shows this mapping where Route ID comprises two fields, that is route ID of interest which is the same as given in Table A-14 in Appendix A3 and D01 and D02 represent the two directions. Node represents the starting node for a given link in a route along with the latitude and longitude of this node.

Figure A-35: Link details for Sydney road network

from	to	capacity (veh/hr)	length (m)	Free Flow Time (min)	Speed Limit (kmph)
1838825566	1832386626	1800	28.199	0.028199	60
1837573347	1832386626	900	372.014	0.4464168	50
1832386628	60901426	1200	32.223	0.032223	60
1833251883	1832386628	2400	440.345	0.440345	60
60901425	1832386628	1200	23.218	0.0278616	50
6337070259	1811939442	1200	255.949	0.3071388	50
1811939468	1811939442	1200	27.154	0.0325848	50

Figure A-36: Mapping between links and route IDs for Sydney road network

Route ID	Node	coordx	coordy
ID01, D01	1835239733	151.1918743	-33.8130148
ID01, D01	1841029218	151.2038181	-33.8199941
ID01, D01	107003138	151.2069749	-33.8231212
ID01, D01	107005514	151.2123318	-33.8252185
ID01, D01	107005516	151.2136362	-33.826821
ID01, D01	107004770	151.2136378	-33.8305887
ID01, D01	107005522	151.2123372	-33.8327116
ID01, D01	793862280	151.2115981	-33.8342085
ID01, D01	1341926235	151.2109056	-33.8367609
ID01, D01	3921232547	151.2121631	-33.8455339
ID01, D02	1157128447	151.2102426	-33.838798
ID01, D02	1157128468	151.2103118	-33.8384744
ID01, D02	793860992	151.2113309	-33.8339253
ID01, D02	793860993	151.2123629	-33.8320673

A5.2 Processing Methodology

The ARSD method is applied using the above information. The methodology for data processing and analysis is summarised below:

Step 1:	Segregate the Sydney dataset into arterial and freeway routes
Step 2:	Determine the link travel time SD (σ_i) using the calibrated ATAP model (Equations 4.1 or 4.2) separately for all the links forming the arterial and freeway routes
Step 3:	Determine the route travel time SD (σ_r) for each arterial and freeway route using the ARSD formula given in Equation 3.4. Assume $\boldsymbol{\gamma}$ as 0.41 and 0.45 for arterial and freeway respectively (refer to Table 4-6).
Step 4:	Plot the estimated route travel time SD across all arterial and freeway routes
Step 5:	Record findings and make recommendations

A6 Application of StrUE

This appendix presents the results obtained from the scenario testing undertaken on the Sydney test network using StrUE modelling.

Scenario-2: The speed limits on all links decreased by 10kmph

Past research has shown that increased speed limits result in higher crash incidence and severity leading to greater uncertainty in the network and exacerbating congestion events. Therefore, this scenario considers transport authorities reducing speed limits across the study area by 10km/h. However, this would likely have adverse impacts on Travel Time measures at both link level and route level. Table A-18 shows the results for a random sample of 10 links in the network, which highlights the expected increase in travel times after the implementation of the speed reductions. However, there is generally a reduction in the coefficient of variation in travel time, which suggests that less variation about the mean travel time, indicating an improvement in standardised reliability.

Table A-18: Scenario-2 - Percentage change in travel time (TT) metrics for randomly selected links in the network

Route	From Node Coordinates	To Node Coordinates	Suburb	Link Length (m)	% change in Expected TT	% change in SD of TT	% change in CoV of TT
M1 General Holmes Dr	-33.936771, 151.1978781	-33.9338887, 151.2106665	Mascot	1277	17	14	-3
A1 Pacific Highway	-33.8354889, 151.2053255	-33.8276538, 151.2006783	North Sydney	981	28	12	-12
A34 Milperra Road	-33.9360905, 151.0109709	-33.9298806, 150.9908058	Revesby	1991	16	32	13
A40 Botany Road	-33.921096, 151.1967991	-33.9055165, 151.2028326	Alexandria	1869	24	13	-8
A40 New South Head Road	-33.8709482, 151.251437	-33.8724953, 151.2598255	Bellevue Hill	885	26	9	-14
A1 Pacific Highway	-33.8053918, 151.1794154	-33.8104226, 151.1771309	Artarmon	642	24	13	-9
A8 Pittwater Road	-33.7887083, 151.2652612	-33.7955396, 151.2530601	Balgowlah	1577	20	-10	-25
A40 Victoria Road	-33.8166083, 151.1093996	-33.8160805, 151.1039224	Ryde	547	21	0	-17
A1 Princes Highway	-34.0238036, 151.0890585	-34.0266529, 151.0850233	Kareela	500	16	18	1
A44 The Horsley Dr	-33.8886837, 150.9695904	-33.8846809, 150.9674909	Villawood	495	22	-5	-22

Figures A-37, A-38 and A-39 show the percentage change in Expected, SD, and CoV of travel times from different zones to the CBD respectively. The expected travel times increased by 18% to 27%, whereas the standard deviations increased by 8% to 68%.

Figure A-37: Scenario-2 - Percentage Increase in expected TT from different zones to CBD

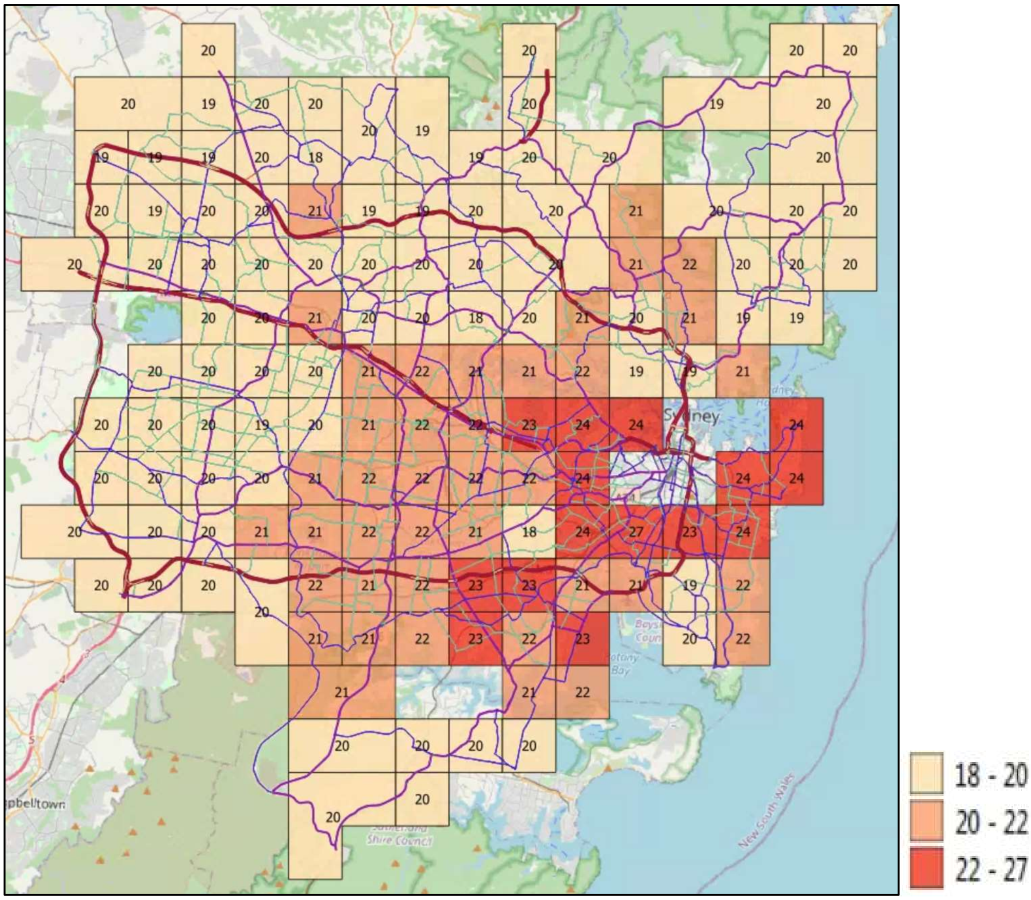


Figure A-38: Percentage Increase in SD of TT from different zones to CBD

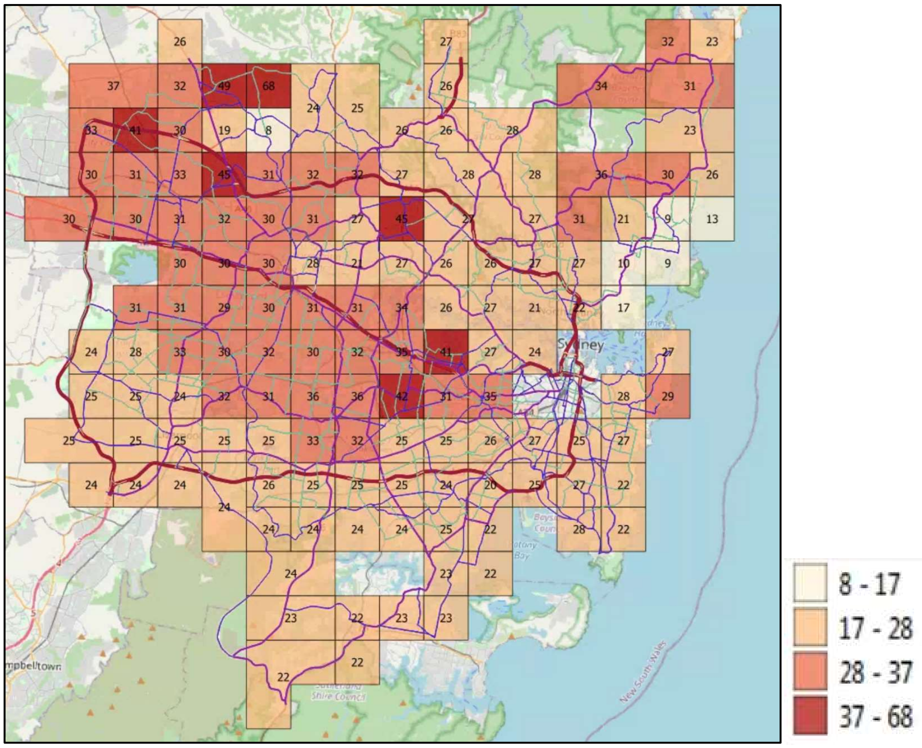
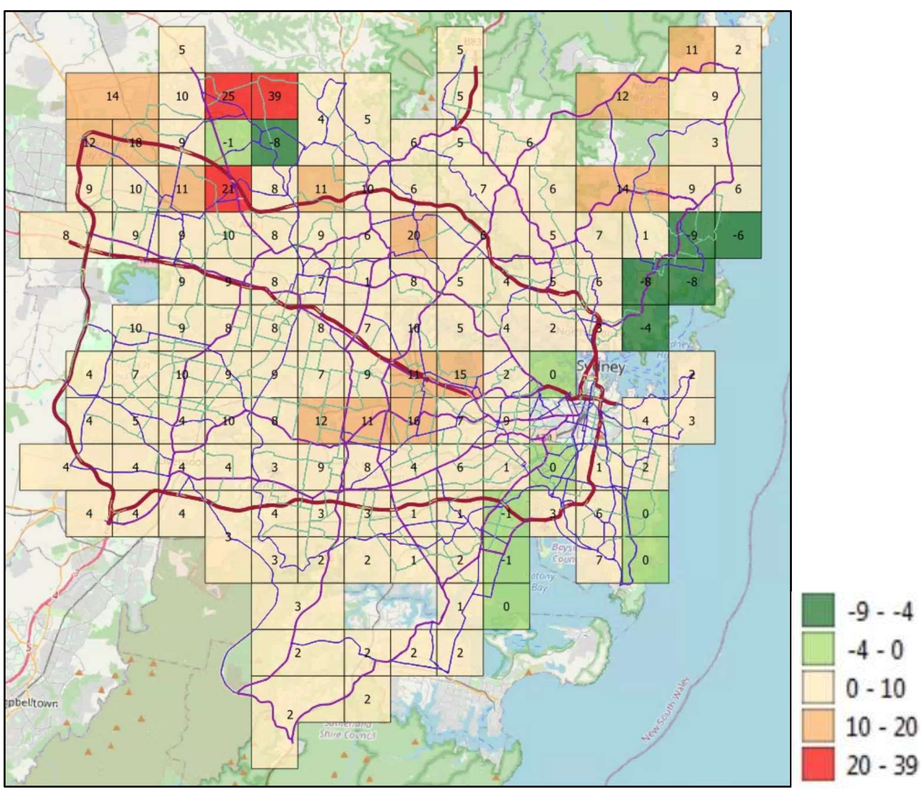


Figure A-39: Percentage change in CoV from different zones to CBD



Note: +ve indicates increase and -ve decrease

Scenario-3: Capacity increase of 25%, both directions, A34, Milperra Road

A34 (Milperra Road), a 26km arterial road connects Bankstown, located in the Inner West Region of Sydney to Newtown, a suburb less than 5km from Sydney's CBD. In this scenario, the capacity of all links on both directions of Milperra Road were increased by 25%. This scenario represents a realistic infrastructure project that traffic authorities could consider as a means of improving road network performance. Table A-19 presents the results of 10 randomly selected links in the network presenting a mixed set of results. Milperra Road and the Pacific Highway present considerable improvement in both travel time and travel time reliability which could be a result of these links serving a metropolitan regional connection between the south-west and north east of Sydney.

Table A-19: Scenario-3 - Percentage change in TT metrics for randomly selected links in the network

Route	From Node Coordinates	To Node Coordinates	Suburb	Link Length (m)	% change in Expected TT	% change in SD of TT	% change in CoV of TT
M1 General Holmes Dr	-33.936771, 151.1978781	-33.9338887, 151.2106665	Mascot	1277	2	-12	-13
A1 Pacific Highway	-33.8354889, 151.2053255	-33.8276538, 151.2006783	North Sydney	981	-4	-39	-37
A34 Milperra Road	-33.9360905, 151.0109709	-33.9298806, 150.9908058	Revesby	1991	-10	-32	-25
A40 Botany Road	-33.921096, 151.1967991	-33.9055165, 151.2028326	Alexandria	1869	3	-14	-17
A40 New South Head Road	-33.8709482, 151.251437	-33.8724953, 151.2598255	Bellevue Hill	885	0	-6	-6
A1 Pacific Highway	-33.8053918, 151.1794154	-33.8104226, 151.1771309	Artarmon	642	-1	0	2
A8 Pittwater Road	-33.7887083, 151.2652612	-33.7955396, 151.2530601	Balgowlah	1577	3	-8	-11
A40 Victoria Road	-33.8166083, 151.1093996	-33.8160805, 151.1039224	Ryde	547	-1	8	9
A1 Princes Highway	-34.0238036, 151.0890585	-34.0266529, 151.0850233	Kareela	500	0	16	16
A44 The Horsley Dr	-33.8886837, 150.9695904	-33.8846809, 150.9674909	Villawood	495	-2	-2	0

Figures A-40 and A-41 show the percentage change in travel time metrics for inbound and outbound links along the Milperra Road. All these metrics show reductions for most of the links except for a few where there is a sharp increase in SD and CoV of travel times. This presents the complex feedback effects capacity alterations to select links have across the network, emphasising the importance of developing accurate and robust network models that capture reliability at both a macro and micro scale. Overall, the inbound route of Milperra Road towards the CBD showed a reduction of 10% in travel time and an increase of 2% in standard deviation. On the other hand, the outbound route showed a reduction of 8% in expected travel time and even greater reduction of 21% in standard deviation. This is intuitive as during the AM peak period, the inbound route services peak traffic demands resulting in more moderate benefits.

Figure A-40: Scenario-3 - Percentage change in travel time metrics for all links along Milperra road (towards city)

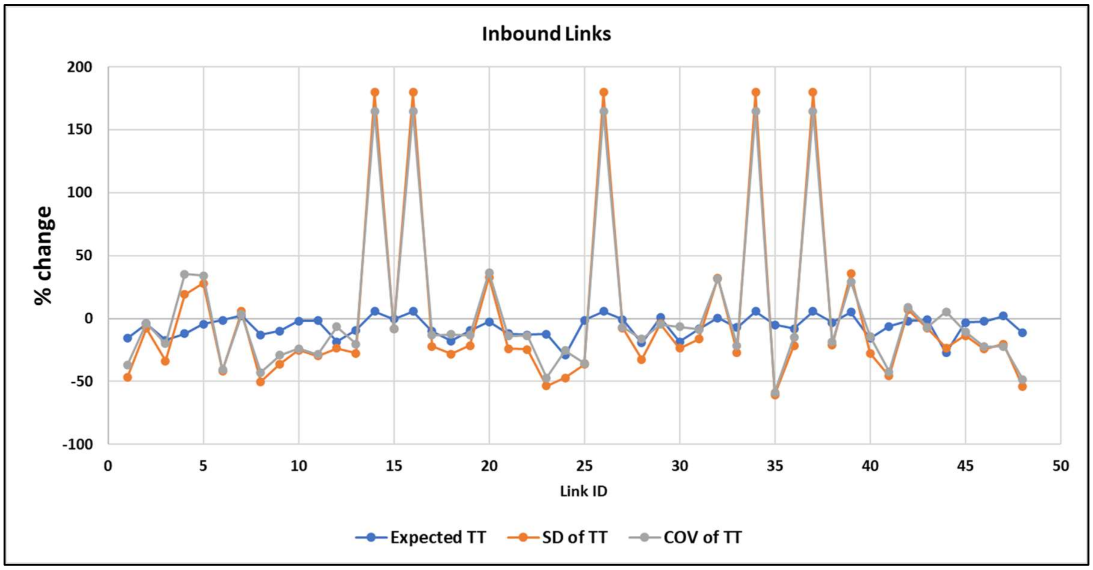
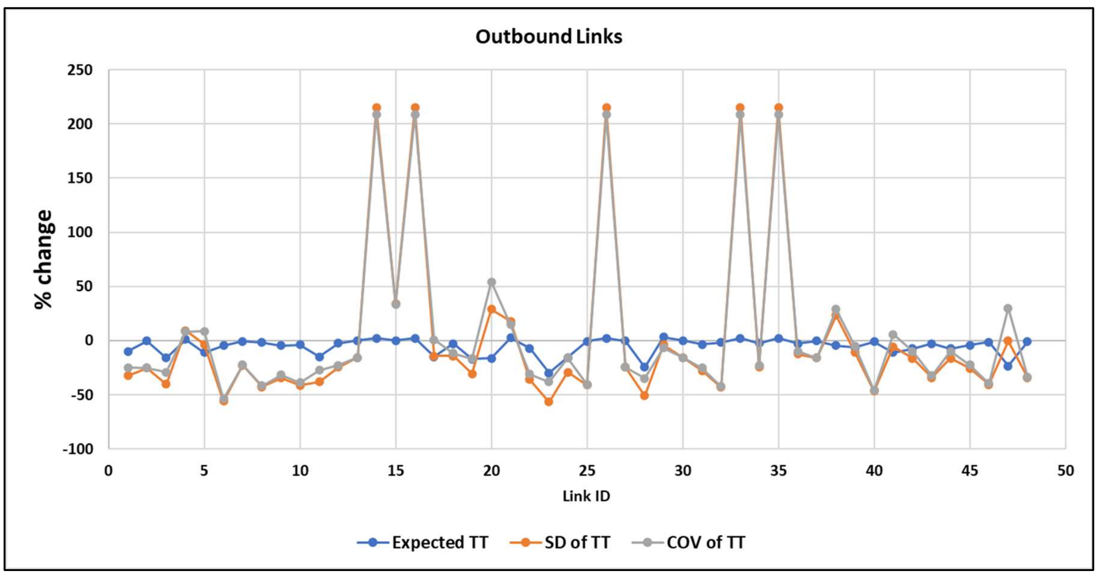
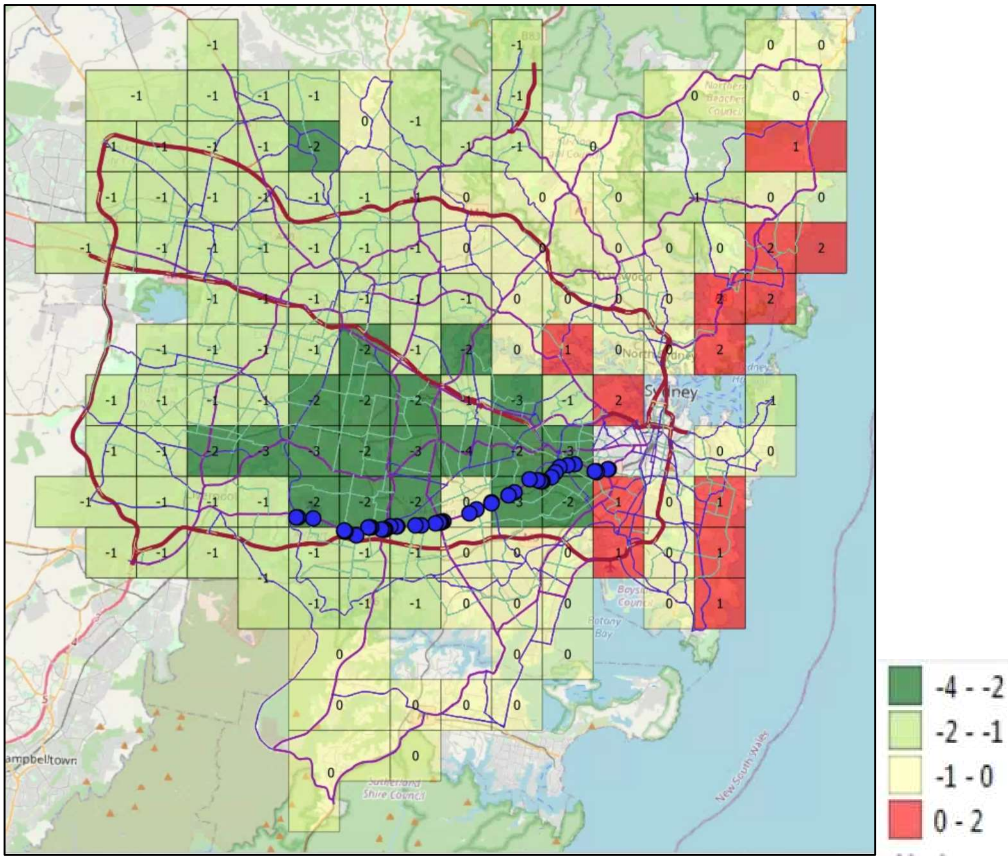


Figure A-41: Scenario-3 - Percentage change in travel time metrics for all links along Milperra road (away from city)



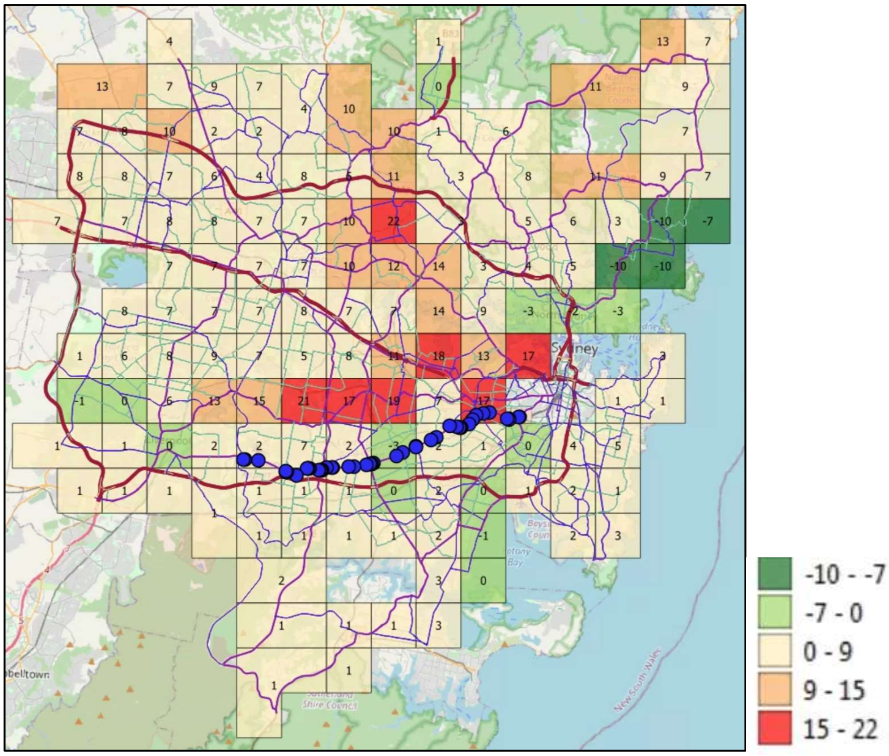
Figures A-42, A-43 and A-44 show the percentage change in Expected, SD, and CoV of travel times from different zones to the CBD respectively. Milperra Road is also highlighted (the blue dots) in these Figures. The expected travel times changed by -4% (improvement) to 2% (deterioration), whereas the standard deviations changed by -10% (improvement) to 22% (deterioration). Although this scenario resulted in a reduction in expected travel times around the zones that the Highway passes through, there is no significant reduction in standard deviation. It is important to note that several zones in the North Shore region showed increases in expected travel times but reduction in standard deviations again positing the notion of an improvement in standardised reliability. On the other hand, several zones in the North West region (around the suburbs of Eastwood, Epping, and Ryde) showed the opposite trend of a reduction in expected travel times combined with a significant increase in standard deviation, highlighting greater unreliability levels.

Figure A-42: Scenario-3 – Percentage change in expected TT from different zones to CBD



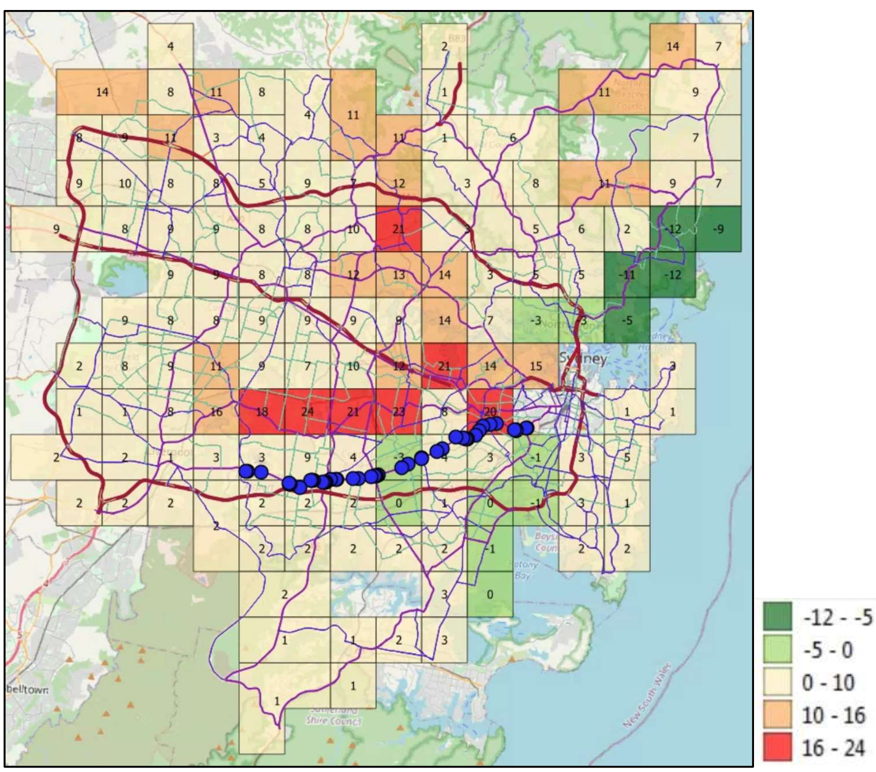
Note: +ve value indicates increase and -ve decrease

Figure A-43: Scenario-3 – Percentage change in SD of TT from different zones to CBD



Note: +ve value indicates increase and -ve decrease

Figure A-44: Scenario-3 - Percentage change in CoV of TT from different zones to CBD



Note: +ve value indicates increase and -ve decrease

The above scenario analysis highlights the fine-grained network wide analysis that can be completed by applying the StrUE traffic assignment approach within a strategic model. This methodology offers direct estimation of standard deviation of travel time for each link and route of the network, thus providing a clear path to measuring and monitoring reliability. The scenario analysis clearly indicates that policies and infrastructure projects do not have homogenous impacts across a network or even within a sub-network. This emphasises the importance of network analysis that endogenously incorporates reliability within the modelling framework.

References

AAA, 2018, 'Road congestion in Australia' by Journey Analytics and The Australian Automobile Association, Canberra, ACT.

Arup 2003, ‘Frameworks for modelling the variability of journey times on the highway network’ by Bates, J, Fearon, J, & Black, L, Arup, London, UK.

Asakura, Y & Kashiwadani, M 1991, ‘Road network reliability caused by daily fluctuation of traffic flow’, PTRC summer annual meeting, 19th, 1991, University of Sussex, UK, pp. 73-84.

Austroads 2007, ‘National Performance Indicators for Network Operations’, by Troutbeck, R, Su, M & Luk, J, AP-R305-07, Austroads, Sydney, NSW.

Austroads 2011a, ‘Valuation of travel time reliability: a review of current practice’, by Tsolakis, D, Tan, F, Makwasha, T & Shackleton, JC, AP-R391-11, Austroads, Sydney, NSW.

Austroads 2011b, ‘Implementation of NPIs to measure congestion’, by Green, D, IR201-11, Austroads, Sydney, NSW.

Austroads 2015, ‘Guide to Traffic Management – Part 2’, Austroads, Sydney, NSW.

Austroads 2016, ‘Congestion and reliability review’, by Deloitte Touche Tohmatsu Ltd, AP-R534-16, Austroads, Sydney, NSW.

Austroads 2019, ‘Infrastructure risk rating manual for Australian roads’, by Zia, H, Harris, D & Smith, D, AP-R587A-19, Austroads, Sydney, NSW.

Bates, J, Polak, J, Jones, P, Cook, A 2001, ‘The valuation of reliability for personal travel’, Transportation Research Part E, vol. 37, pp. 191 -229.

Bureau of Public Roads 1964, Traffic assignment manual, US Department of Commerce, Washington, DC, USA.

Chang, Y, & Edara, P 2017, ‘A Hybrid Origin-Destination Trip Matrices Estimation Model Using Machine Learning Techniques’, Annual Meeting of Transportation Research Board. Available at: http://onlinepubs.trb.org/onlinepubs/conferences/2017/CensusData/ChangPoster.pdf

Chen, A, Zhou, Z & Lam, WH 2011, ‘Modelling stochastic perception error in the mean-excess traffic equilibrium model’, Transportation Research Part B: Methodological, vol. 45, no. 10, pp. 1619-40.

Chiu, YC, Bottom, J, Mahut, M, Paz, A, Balakrishna, R, Waller, T, Hicks, J 2011. ‘Dynamic traffic assignment: A primer’.

Chorus, CG 2012, ‘Regret theory-based route choices and traffic equilibria’, Transportmetrica, vol. 8, no. 4, pp. 291-305.

Clark, S & Watling, D 2005, ‘Modelling network travel time reliability under stochastic demand’, Transportation Research Part B: Methodological, vol. 39, no. 2, pp. 119-40.

Dixit, V, Gardner, LM & Waller, ST 2013, ‘Strategic user equilibrium assignment under trip variability’, Transportation Research Board annual meeting, 92nd, 2013, Washington, DC, USA, TRB, Washington, DC, USA.

Emam, EB& Al-Deek, H, 2006, ‘Using real-life loop detector data to develop new methodology for estimating freeway travel time reliability’, Transportation Research Record, no. 1959, pp 140-50.

Fosgerau M., Hjorth K. and Fukuda. D., 2008, ‘Travel time variability: Definition and valuation’, accessed online 16 December 2019 at https://www.researchgate.net/publication/264967313_Travel_time_variability_Definition_and_valuation

Gao, S, Frejinger, E & Ben-Akiva, M 2010, ‘Adaptive route choices in risky traffic networks: a prospect theory approach’, Transportation Research Part C: Emerging Technologies, vol. 18, no. 5, pp. 727-40.

Gupta, S, Vovsha, P, Dutta, A, Livshits, V, Zhang, W & Zhu, H 2018, ‘Incorporation of travel time reliability in regional travel model’, Transportation Research Record, no. 2672, pp. 46-57.

Hall, M, & Gaffney, J, 2018, ‘Melbourne Managed Motorways – Traffic Science for Optimisation’ 28th ARRB International Conference, Brisbane, Qld

Karl, CA & Bechervaise, N 2004, ‘Driving home the advantage: innovative ITS for toll roads', World Congress on ITS, 11th, 2004, Nagoya, Aichi, Japan, ITS Japan, Tokyo, Japan.

Kouwenhoven, M & Warffemius, P 2017, ‘Forecasting travel-time reliability in road transport: A new model for the Netherlands’, ITF Roundtable Reports Quantifying the Socio-economic Benefits of Transport, vol 1, p.57.

Li, Q, Liao, F, Timmermans, HJ & Zhou, J 2016, ‘A user equilibrium model for combined activity–travel choice under prospect theoretical mechanisms of decision-making under uncertainty’, Transportmetrica A: Transport Science, vol. 12, no. 7, pp. 629-49.

Lo, HK, Luo, XW & Siu, BW 2006, ‘Degradable transport network: travel time budget of travellers with heterogeneous risk aversion’, Transportation Research Part B: Methodological, vol. 40, no. 9, pp. 792-806.

MRWA 2016, ‘Journey time reliability indicators, Desktop review’, by WSP Parson Brinckerhoff, Main Roads Western Australia, Perth, WA.

Mishra, S, Tang, L, Ghader, S, Mahapatra, S & Zhang, L 2018, ‘Estimation and valuation of travel time reliability for transportation planning applications’, Case Studies on Transport Policy, vol. 6, no. 1, pp. 51-62.

National Academy of Sciences, 2000, ‘Highway Capacity Manual’, Transportation Research Board, National Research Council, USA, Access online 16 December 2019 at https://sjnavarro.files.wordpress.com/2008/08/highway_capacital_manual.pdf

NZTA 2013, ‘Economic evaluation manual’, New Zealand Transport Agency, Wellington, New Zealand.

Nicholson, A., 2015, ‘Travel time reliability benefits: Allowing for correlation’, Research in Transportation Economics, vol. 49, pp.14-21.

Nie, YM 2011, ‘Multi-class percentile user equilibrium with flow-dependent stochasticity’, Transportation Research Part B: Methodological, vol. 45, no. 10, pp. 1641-59.

Osterle, I., Gort, W. & Weiss, A 2017. Forecasting and Valuing Travel Time Variability for Cost-Benefit Analysis. Road & Transport Research: A Journal of Australian and New Zealand Research and Practice, 26, 21.

Ou, J., Lu, J., Xia, J., An, C. & Lu, Z 2019. Forecasting and Valuing Travel Time Variability for Cost-Benefit Analysis. IEEE Access, 7, doi: 10.1109/ACCESS.2019.2901289.

PIARC, 2019, Journey Time and Travel Time Reliability, Technical Committee A.2 Road Transport System and Economic and Social Development, report 2019 R35EN, World Road Association (PIARC), access online 14 December 2019 at https://www.piarc.org/ressources/publications/11/85139d3-31456-2019R35EN-Journey-Time-and-Travel-Reliability.pdf

Prashker, JN & Bekhor, S 2004, ‘Route choice models used in the stochastic user equilibrium problem: a review’, Transport Reviews, vol. 24, no. 4, pp. 437-63.

Moylan, E, Jian, S, Wijayaratna, K, & Waller, ST 2018, ‘Journey time reliability model development’, by Moylan, E, Jian, S, Wijayaratna, K and Waller ST, unpublished report, Research Centre for Integrated Transport Innovation, University of New South Wales, Sydney, NSW.

Shao, H, Lam, WH & Tam, ML 2006, ‘A reliability-based stochastic traffic assignment model for network with multiple user classes under uncertainty in demand’, Networks and Spatial Economics, vol. 6, pp. 173-204.

Sun, C, Cheng, L & Ma, J 2018, ‘Travel time reliability with boundedly rational travellers’, Transportmetrica A: Transport Science, vol. 14, no. 3, pp. 210-29.

SYSTRA 2015, ‘Value of Travel Time Savings – Peer Review & Audit’, by Le Masurier, P. Polak, J. Pawlak, J. Fosgerau, M. Hjorth, K, SYSTRA LIMITED, Reference number 103217, UK.

Szeto, W, O'Brien, L & O’Mahony, M 2007, ‘Generalisation of the risk-averse traffic assignment’, Transportation and Traffic Theory, vol. 17, pp. 127-53.

Taylor, MA 2013, ‘Travel through time: the story of research on travel time reliability’, Transportmetrica B: Transport Dynamics, vol. 1, no. 3, pp. 174-94.

Transport and Infrastructure Council, 2016, ‘A scoping study for future research into values of travel time savings for project appraisal and toll road patronage forecasting’, 2014 National Guidelines for Transport System Management in Australia, Transport and Infrastructure Council, Australia.

Uchida, K 2014, ‘Estimating the value of travel time and of travel time reliability in road networks’, Transportation Research Part B: Methodological, no. 66, pp. 129-47.

Wu, X 2015, ‘Study on mean-SD shortest path problem in stochastic and time-dependent networks: a stochastic dominance based approach’, Transportation Research Part B: Methodological, vol. 80, pp. 275-90.

TRANSPORT AND INFRASTRUCTURE COUNCIL